PHY1201 General Physics Ⅰ - Notes

Ch1 Vectors and Simple Calculus

[Chapter 1 PDF]

Physical Quantities (P10)

Scalar 矢量

- Numbers Only

Vector 标量

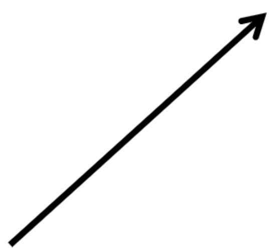
- Consists of **scalar magnitude + direction**
- $\vec{A}$
- magnitude = $|A|$

Physical quantities in vector

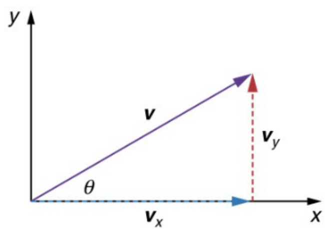
- displacement x 位移
- velocity v、momentum p
- angular velocity ω、angular momentum $\mathbf{L}$ 角动量
- acceleration a、force F
- **electric field E, magnetic field B**

Representing Vector Graphically (P14)

- a vector is represented graphically by an arrow
- The *length* of the arrow is the vector's *magnitude*.
- The *direction* of the arrow is the vector's *direction*.



- Representing vector using numbers
- Magnitude can be represented by a number, say v in the figure
- Direction can represented by the angle made with a fixed direction, angle θ made with the x-axix



Calculation of Vector (P15)

Addition

tail(A)->head(A) coincidences with tail(B)->head(B)

- **Head to tail method**
- Parallelogram method

Subtraction (P20)

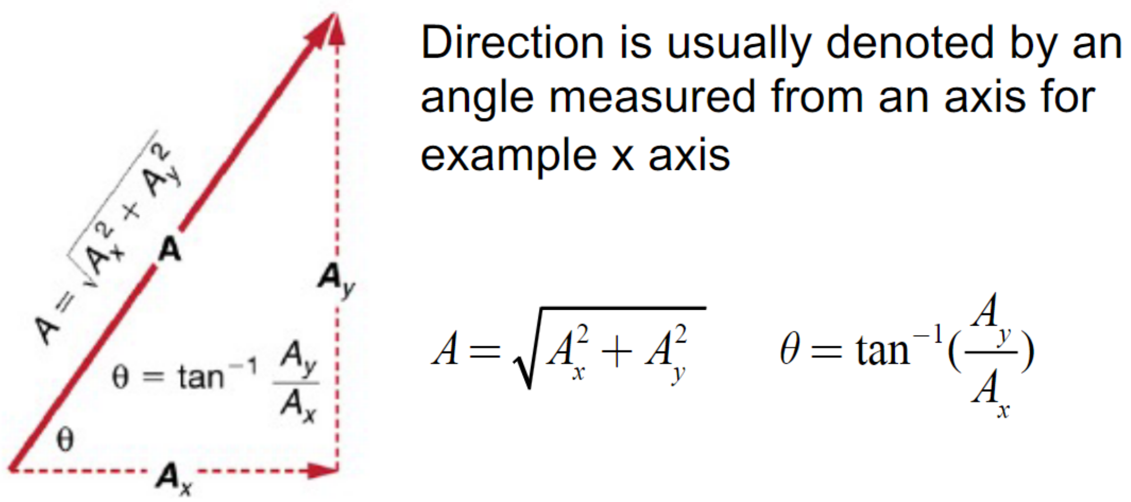
$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

Scalar Product (P21) 数量积

- $\left|c\left|\vec{A}\right|\right| = c\left|\vec{A}\right|$
- Direction remains if $c > 0$, or opposite if $c < 0$

Vector Component

- $\vec{A}_x = \vec{A}\cos\theta$
- $\vec{A}_y = \vec{A}\sin\theta$
- $\vec{A} = \vec{A}_x + \vec{A}_y$



Unit Vector (P28)

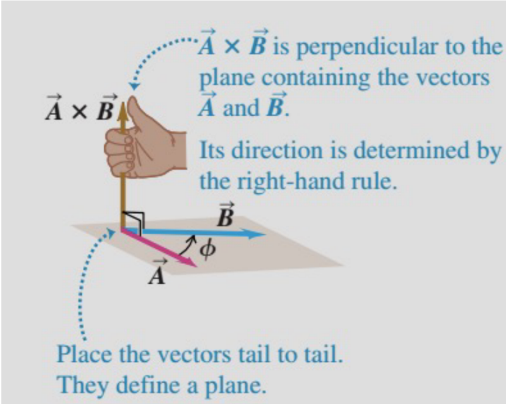
- $\left|\hat{i}\right| = \left|\hat{j}\right| = 1$
- $\vec{C} = A_x\hat{i} + A_y\hat{j}$
- $\vec{A}_x = A_x\hat{i}; \vec{A}_y = A_y\hat{j}$

Scalar Product (P55) 点积

- also dot product
- $\vec{A} \cdot \vec{B} = |A|\,|B|cos\phi$
- $\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y$

Vector Product (P57) 叉积

- The vector product (“cross product”) of two is a vector .
- The product is a vector $\vec{C} = \vec{A} \times \vec{B}$
- $\vec{C}$ has magnitude $|\vec{A} \times \vec{B}| = AB \sin \phi$
- $\vec{C}$ is perpendicular to $\vec{A}$ *and* $\vec{B}$ and the *right-hand rule* gives its direction. See the figure



Ch2 Motion

[Chapter 2 PDF]

Definition (P3)

Motion

> change of **position** with **time** (position as function of time)

Mechanics 动力学

> study of motion and the cause of motion

Position

> coordinates or position vector, like:
>
> - x
> - $\vec{r} = x\hat{i} + y\hat{j}$
> - $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Displacement & Average Velocity

> - $\Delta x = x_2 - x_1$
> - $v_{\mathrm{av}-x} = \frac{\Delta x}{\Delta t}$
> - $\iff \quad x_2 = x_1 + v_{\mathrm{av}-x}\, \Delta t$
> - Average velocity is defined for a **time interval**

Instantaneous velocity

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{\mathrm{d}x}{\mathrm{d}t}$$

- on $x - t$ graph, v_x is the slope of the **tangent** to the curve

Average & Instantaneous acceleration

$a_{\text{av}-x} = \frac{\Delta v_x}{\Delta t}$

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{\mathrm{d} v_x}{\mathrm{d} t}$$

- on $v - t$ graph, a_x is the slope of the tangent to the curve

Constant Acceleration Motion (P13)

$v - t$ graph

- **straight line**
- The area under $v - t$ graph is Δx, proof:

$$\Delta x_i = v_x \Delta t_i \Rightarrow \sum v_x \Delta t = \Delta x$$

Equations

> - $v_x = v_{0x} + a_x t$
> - $x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$
> - $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$
> - $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right) t$

2D and 3D Motion (P30)

Velocity under vector

$\vec{v}_{\text{av}} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t} \hat{i} + \frac{\Delta y}{\Delta t} \hat{j} + \frac{\Delta z}{\Delta t} \hat{k}$

other definition likewise

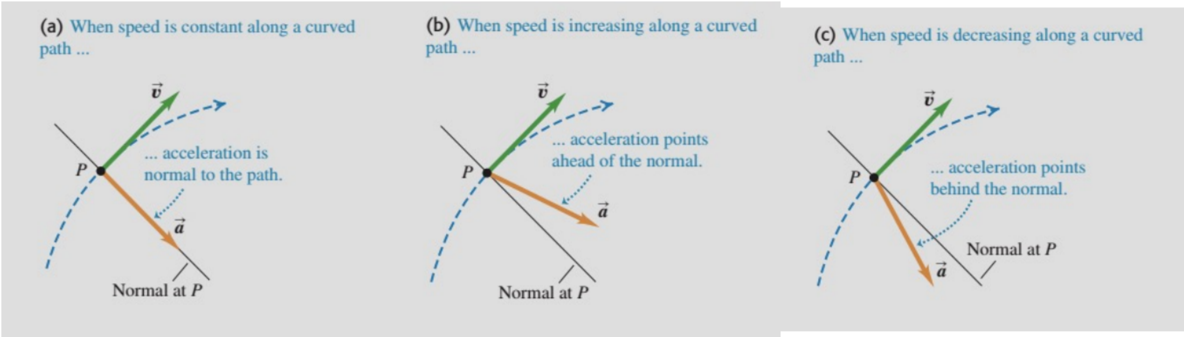
- motion(position-time function)
- instantaneous velocity
- average acceleration
- instantaneous acceleration

Difference between Insta. Speed and Velocity

- speed (速率) is **scalar**
- velocity is **vector**

Insta. Velocity and Acceleration

- Instantaneous velocity is always **tangent to the path**
- Instantaneous acceleration
  - normal = perpendicular
  - ahead = higher speed pull outward & forward



Projectile Motion (P56)

- Gravity only (ignoring air resistance)
- **The x and y motion are separable**

Equations

> - $x = v_0 \cos\theta_0 t$
> - $y = v_0 \sin\theta_0 t - \frac{1}{2}gt^2$
> - $v_x = v_0 \cos\theta_0$
> - $v_y = v_0 \sin\theta_0 - gt$

Uniform circular motion (P69)

> - $a = \frac{v^2}{r}$
> - $a_{\text{rad}} = \frac{4\pi^2 r}{t^2}$
> - $\Delta\theta = \frac{\Delta s}{\Delta r} = \frac{v\Delta t}{\Delta r}$

Exercise 2

Thinking Questions

Uniform Ellipse Motion

- When rounding ellipse (椭圆) or other curves, the **acceleration is always perpendicular with velocity**

Same Height with Different Initial Speed

A zookeeper fires a tranquilizer dart directly at a monkey. The monkey lets go at the same instant that the dart leaves the gun barrel. The dart reaches a maximum height *P* before striking the monkey. Ignore air resistance.

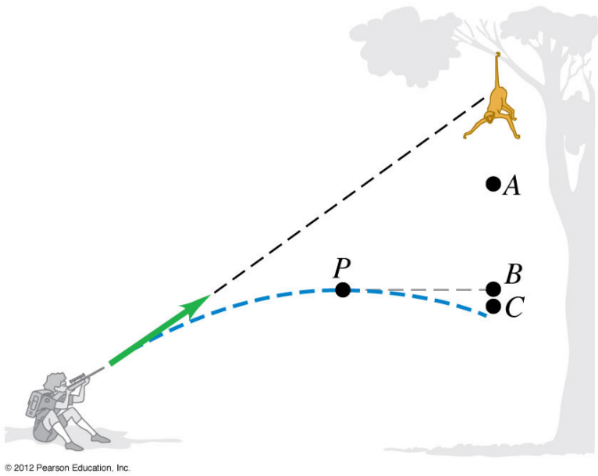
When the dart is at *P*, the monkey

A. is at *A* (higher than *P*).

B. is at *B* (at the same height as *P*).

C. is at *C* (lower than *P*).

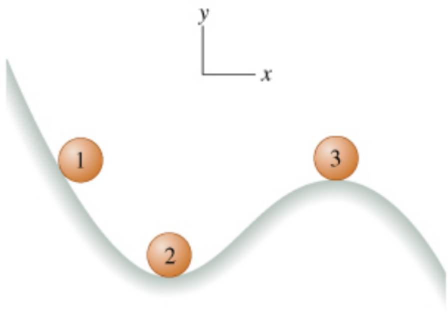
D. not enough information given to decide



> Assume there is no g and monkey don't move. The dart with an initial speed v_x, v_y will hit the monkey at time T.
>
> So monkey's coordinates are (v_xT, v_yT).
>
> Now with gravity, the coordinates at time t are:
>
> $$\begin{aligned} x_{\text{dart}}(t) &= v_x t \\ y_{\text{dart}}(t) &= v_y t - \frac{1}{2}gt^2 \\ x_{\text{monk}}(t) &= v_x T \\ y_{\text{monk}}(t) &= v_y T - \frac{1}{2}gt^2 \end{aligned}$$
>
> Finally at time T we have:
> $x_{\text{dart}}(T) = x_{\text{monk}}(T) = v_xT,\ y_{\text{dart}}(T) = y_{\text{monk}}(T) = v_yT - \frac{1}{2}gT^2$
>
> Thus dart will still **hit monkey at time T**.
>
> When dart reaches P, $v_y = 0$. At this time monkey's $v_y > 0$. So if monkey's height is no greater than P, it will drop quicker and dart cannot hit monkey.
>
> Thus monkey is higher than P.

Tutorial 2

Acceleration Direction



- Point 1: velocity increasing, direction not changed
  - acceleration downward to the right
- Point 2: velocity right, velocity **at next moment** lifted up
  - acceleration straight up
- Point 3: acceleration straight down
- acceleration in circular uniform always point to **the center of the circle**
- as $a = \frac{v^2}{r}$, smaller r, greater a when v is given.

Projectile Motion

- When thrown at 45° with respect to the horizontal, the object fly the longest distance
- When $\theta > 45^\circ$, greater θ, shorter distance
- greater θ, greater v_y, means:
  - higher **maximum height**
  - longer **time**

Lab: Projectile Motion

$$\begin{aligned} \because x &= x_0 + v_x\, t \\ v_x &= v_0 \cos\theta \\ \therefore x - x_0 &= v_0\, t \cos\theta \\ \Leftrightarrow v_0 &= \frac{x - x_0}{t\cos\theta} \end{aligned}$$

Ch3 Newton's law and Force

[Chapter 3 PDF]

Newton's Law (P3)

Characteristics of Force

- Forces causes **change in the motion**
- Force is an interaction between **two objects**
- Force as a **vector** has direction and magnitude

Newton's First Law

> If there is no force or **no net force** on a body the body moves with **constant velocity** and zero acceleration or remain at **rest**
>
> $$\Sigma\vec{F} = 0 \Rightarrow \vec{v} = C,\ \vec{a} = 0$$

Newton's Second Law

> $$\begin{aligned} \vec{a} &\propto \Sigma\vec{F} \\ |a| &\propto \frac{1}{m} \\ \therefore \vec{a} &= \frac{\Sigma F}{m} \\ \Leftrightarrow \vec{F} &= m\vec{a} \\ 1\mathrm{N} &= 1\mathrm{kg}\cdot\mathrm{m/s}^2 \end{aligned}$$

Mass

> - Mass is proportional to the amount of matter
> - Mass resists motion, tendency to stay unmoved.
> - Larger mass ↔ Larger motion resistance ↔ Smaller acceleration.

Newton's Third Law

> - If you exert a force (**action**) on an object, the object always exerts a force (the **reaction**) back upon you.
> - A force and its reaction force have the same magnitude but **opposite directions**. These forces act on **different bodies**.

Common Forces

- The normal force (surface -> object): **perpendicular** to the surface. Contact force.
- Friction force (surface -> another surface): **parallel** to the surface. Contact force.
- Tension force: via rope or cord. Contact force.
- Weight: Long-range force.
- Push: Contact force.

Friction

- **kinetic friction**: relative motion, $f_k = \mu_k N$ is **static**
- **static friction**: no relative motion, $0 \leq f_s = \Sigma F \leq \mu_s N$ is **proportional**
- $f_s = F_{\text{eternal}}$

(De)composition of Force

Free-body Diagram / Force Diagram 隔离体图

- Normal force must $\perp$ the surface.
- **There is no $m\vec{a}$ force**.

Dynamics of circular motion

$$v = \frac{2\pi R}{T}, a = \frac{v^2}{R} = \frac{4\pi^2}{T^2}R = \omega^2 R$$

With no net force as **centripetal force**, object will go linear uniform motion

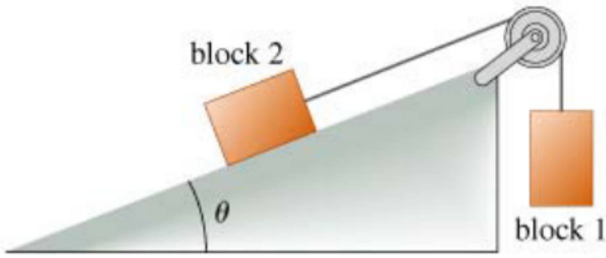
Exercise 3

Application of Newton's law (P41)

- When rounding a flat curve with radius R, $v_max = \sqrt{\mu gr}$
- When sliding down the slope with **no friction**, $a = g\sin\alpha$.
- When sliding down the slope with **zero acceleration**, $\mu = \tan\alpha$.

Tutorial 3

Slope



> - Given: frictionless slope, block 1 pulling 2 up at a constant acceleration
> - let direction going down along slope be x, perpendicular down to slope be y, and vertically down be g. (We can have different directions for different objects.)
> - If have a friction down the slope, the answer will become $a_{1g} = \frac{m_2 g\sin\theta - \mu m_2 g\cos\theta - m_1 g}{m_1 + m_2}$
>
> $$\begin{aligned} a_{2x} < 0, a_{1g} > 0 \\ T_1 = T_2 \Leftrightarrow a_{2x} = -a_{1g} \\ T - m_1 g = m_1 a_{1g} \\ T - m_2 g\sin\theta = m_2 a_{2x} \\ \Rightarrow m_2 g\sin\theta + m_2 a_{2x} = m_1 g + m_1 a_{1g} \\ \Rightarrow a_{1g} = \frac{m_2 g\sin\theta - m_1 g}{m_1 + m_2} \end{aligned}$$

Slope Variant (Assignment 4)

> - Given: slope kinetic friction coefficient μ; $m_1 = m_2$
>
> $$mg - T = ma(a > 0)$$
> $$T - \mu mg\cos\theta - mg\sin\theta = ma$$
> $$T = \frac{1}{2}mg(1 + \mu\cos\theta + \sin\theta)$$
> $$a = \frac{mg - T}{m} = \frac{1}{2}g(1 - \mu\cos\theta - \sin\theta)$$

Rope

Let a rope's right pull be F_R and let pull be F_L. So $F_R - F_L = ma_{\text{rope}}$

$F_R = F_L$ in case that:

- the rope is massless ($m = 0$)
- **the rope is moving at constant speed** ($a_{\text{rope}} = 0$)

Pull - Friction

- $F_1 - \mu mg = ma \Leftrightarrow F_1 = \mu mg + ma$
- $F_2 - \mu mg = m(2a) \Leftrightarrow F_2 = \mu mg + 2ma \rightarrow F_1 < F_2 < 2F_1$
- $F_3 - \mu\frac{m}{2}g = \frac{m}{2}(2a) \Leftrightarrow F_3 = \frac{1}{2}\mu mg + ma \rightarrow \frac{1}{2}F_1 < F_3 < F_1$

Ch4 Work, Kinetic Energy, Potential Energy

Chapter 4 PDF

Work & Kinetic Energy (P6)

Definition

$$F_x\Delta x = \frac{1}{2}mv_x^2 - \frac{1}{2}mv_0^2$$

Where $F_x\Delta x$ is **work done by a force**, and the right is **energy acquired by an object**.

2-Dimensional Work

$$W = \vec{F}\cdot\Delta\vec{s} = Fs\cos\phi = F_{\parallel}\Delta s$$

- $\phi < 90^\circ$, will give the object energy
- $\phi = 90^\circ$, will have no energy transfer
- $\phi > 90^\circ$, will draw energy from the object

Kinetic Energy

$E_K = \frac{1}{2}mv^2$

$$W = F_{\parallel}\Delta s = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = E_{K_{\text{FIN}}} - E_{K_{\text{INIT}}} = \Delta E_K$$

- Force and velocity in the same direction, **positive work**
- Force and velocity in the opposite direction, **negative work**
- Force and velocity are perpendicular, **zero work**

> - In an uniform circular motion, $\vec{F} \perp \vec{v}$ so $W = F\Delta s\cos 90^\circ = 0$. So E_K doesn't change and keep the same value.
> - **Normal force perpendicular to motion in curve**
> - In an accelerated motion, positive work
> - In a friction motion, negative work
> - $\Sigma \vec{F}_i = m\vec{a} \Rightarrow \Sigma W_i = \text{Sum of all work}$

$F - x$ graph

The area under the curve is W.

Potential Energy & Energy Conservation (P27)

Definition

- When an object falls, gravity force do work: $W_{mg} = (-mg)\Delta y = (-mg)(-H) = mgH > 0$
- When you lift the object, your force counter gravity and do work: $W_F = mgH = mg\Delta y = -W_{mg} > 0$

Conversion of E_P to E_K

- Before and after a fall: $0 = \frac{1}{2}mv^2 + mg\Delta y$, where v is final speed, $\Delta y < 0$
- $E_K + mgH = 0 = \text{Constant} \Rightarrow E_P = mgH$
- During the fall, E_P is converted into E_K

$$\begin{aligned}
\because v_{y1}^2 - v_{y0}^2 &= -2g\Delta H \\
v_{x1} &= v_{x0} = v_0 \\
v_1^2 &= v_{x1}^2 + v_{y1}^2 \\
\therefore v_1^2 - v_0^2 &= -2g\Delta H \\
\therefore \frac{1}{2}mv_1^2 - \frac{1}{2}mv_0^2 &= -mg(H_1 - H_0) \\
\Leftrightarrow \frac{1}{2}mv_1^2 + mgH_1 &= \frac{1}{2}mv_0^2 + mgH_0
\end{aligned}$$

Conservation of mechanical energy

- mechanical energy = E_P + E_K
- when only gravity do work (zero other force or **zero other work**), mechanical energy conserved
- $\Delta E_P = -W_G$, when mg do positive work, lose E_P.
- E_P is only dependent on y coordinate.

First Law of Thermodynamics

$$\begin{aligned}
\because \Delta E_P &= -W_G \\
W_f + W_G &= \Delta E_k \\
\therefore W_f &= \Delta E_P + \Delta E_K = \Delta E_M \\
\Leftrightarrow W_f &= -E_{\text{lost}} < 0
\end{aligned}$$

W_f is converted into internal energy (E_K of molecules). Overall energy is conserved.

Elastic Potential Energy

- Because $F = kx$, and $F - x$ area is a triangle
- So $W = \frac{1}{2}kx^2$
- Whether stretching or compressing, you do positive work
- When it is released, it do positive work on object

Spring Mass System

- spring + mass, no friction, start at zero speed
- E_P change in $\cos$, E_K change in $\sin$

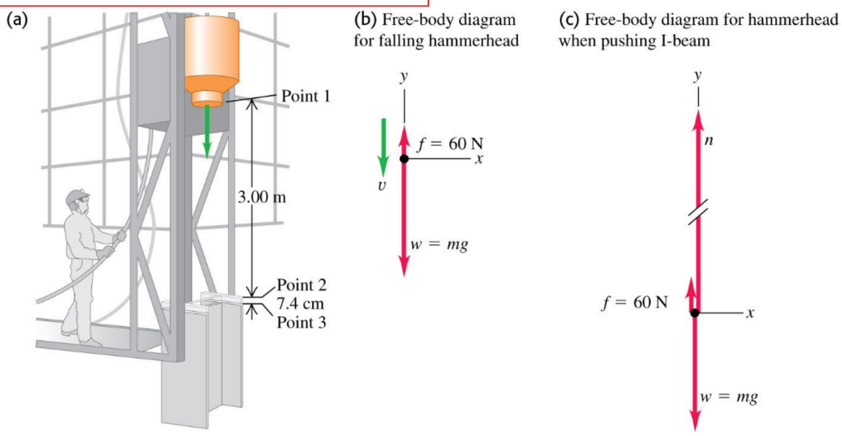
More Potential Energy

- Newtonian Gravitational Potential: $E_{PG} = -G\frac{Mn}{r}$
- Coulomb Electric Potential: $E_{Pe} = k\frac{Qq}{r}$

Exercise 4

Forces on a Hammerhead

The 200-kg steel hammerhead of a pile driver is lifted 3.00 m above the top of a vertical I-beam being driven into the ground (Fig. 6.12a). The hammerhead is then dropped, driving the I-beam 7.4 cm deeper into the ground. The vertical guide rails exert a constant 60-N friction force on the hammerhead. Use the work–energy theorem to find (a) the speed of the hammerhead just as it hits the I-beam and (b) the average force the hammerhead exerts on the I-beam. Ignore the effects of the air.



> - Problem (a): $v = \sqrt{57}\mathrm{m \cdot s^{-1}}$
> - Problem (b):
>
> $$W_w - W_f - W_n = 0 - E_K$$
> $$(mg - f - n)s = 0 - \frac{1}{2}mv^2$$
> $$n = mg - f + \frac{mv^2}{2s}$$
> $$= 78927\mathrm{N}$$

Ch5: Rotation Motion

Chapter 5A PDF

What do we Study

- *Rigid Body* rotation of a *Fixed* Axis
- **Axis of Rotation: the line formed by motion center** of all particles
- Axis of Rotation can be outside the object, like: the moon's Axis of Rotation is in the center of the earth

1-Dimensional Motion

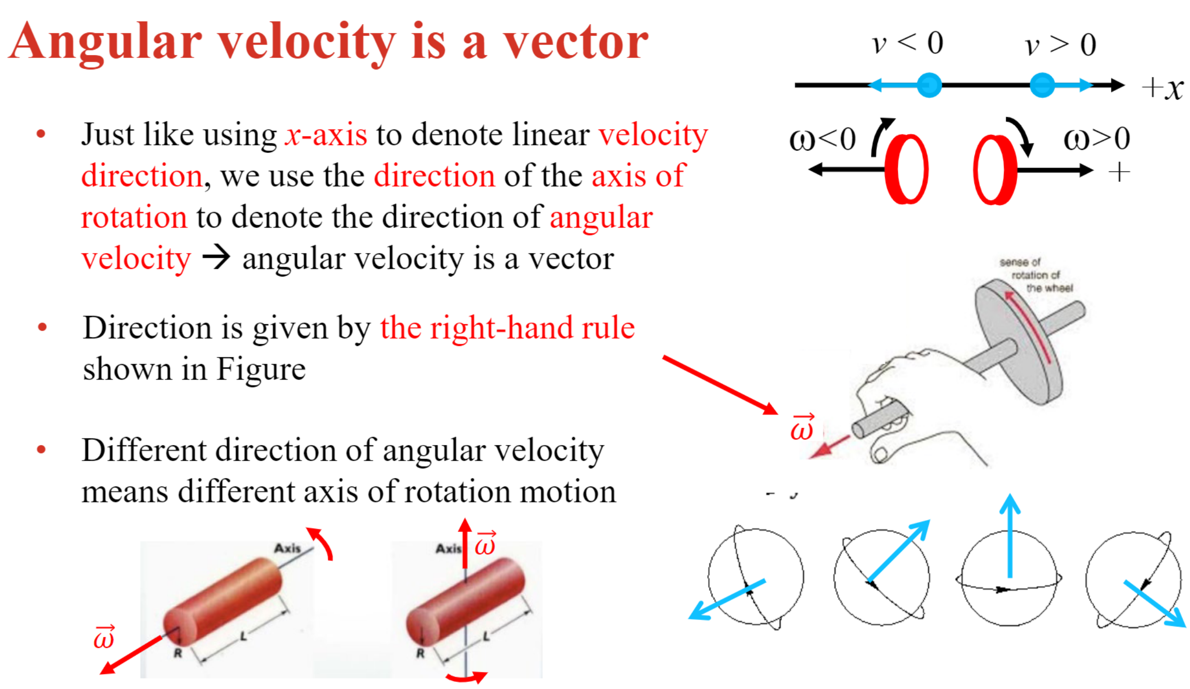
- use the angle θ between $+x$-axis for a coordinate
- $s = \theta r$, where θ is radian, $360^\circ = 2\pi$ rad

Angular θ(coordinate), ω(velocity), α(acceleration)

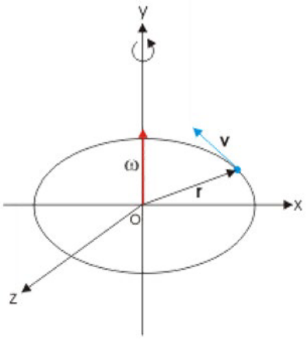
- $\Delta\theta = \theta_2 - \theta_1$
- Clockwise: <0, Counterclockwise: >0
- $\omega_{\text{av-z}} = \Delta\theta/\Delta t$, where z is the axis of rotation
- $\vec{\omega}$ is a vector, Direction given by **right-hand rule**
- $\alpha_z = \frac{\mathrm{d}\omega_z}{\mathrm{d}t} = \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}$ is a vector
- $a_x \Leftrightarrow \alpha_z$, $v_x \Leftrightarrow \omega_z$, $x \Leftrightarrow \theta$, θ **must be the reminder of** 2π

Direction of Angular Quantities

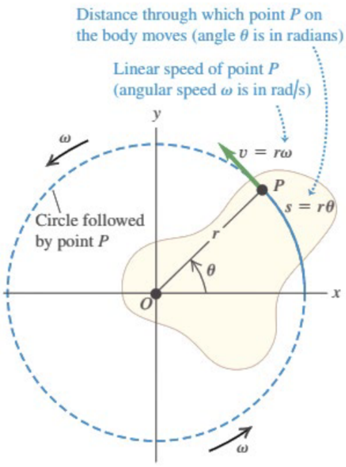
Note: Angular velocity direction **is not tangential velocity**



Relation between angular velocity, position vector and velocity



- Angular velocity is perpendicular to the plane of the circle of motion
- It is perpendicular to velocity $\vec{v}$, as $\vec{v}$ is tangential to the circle
- When $\vec{r}$ is in the plane of the circle, $s = r\theta \quad \rightarrow v = r\omega$



Tangential and radial(=centripetal) acceleration 切向/向心加速度

- $x(t) = r\cos\theta(t), y(t) = r\sin\theta(t)$
- $a_x(t) = -r\omega\sin\theta(t), a_y(t) = r\omega\cos\theta(t)$ (note $\theta'(t) = \omega$)
- **radial** $a > 0$
- **tangential** $a = 0$ **if speed is constant**

$$\because v = r\omega$$
$$\therefore a_r = \frac{v^2}{r} = r\omega^2$$
$$a_t = \frac{\Delta v}{\Delta t} = r\frac{\Delta\omega}{\Delta t} = r\alpha$$

Translational & Rotational Quantities

- $\vec{r} \Leftrightarrow \theta$
- $\vec{v} \Leftrightarrow \omega$
- $\vec{a_{\text{tan}}} \Leftrightarrow \alpha$
- $m \Leftrightarrow I$

Rotational Kinetic Energy (Moment of Inertia) 转动惯量

- $I = \Sigma m_i^2 r_i^2$
- Smaller I makes it easier to rotate
- I varies depend on the rotation axis
- $E_K = \frac{1}{2}mv^2 \Rightarrow E_R = \frac{1}{2}I\omega^2$
- **Total Kinetic Energy = Translational + Rotational** $E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$
  - Translational: The system move externally as a whole
  - Rotational: The objects in system rotates internally

Center of Mass

$x_c = \frac{m_1x_1+m_2x_2}{m_1+m_2}$

When rotating through center of mass

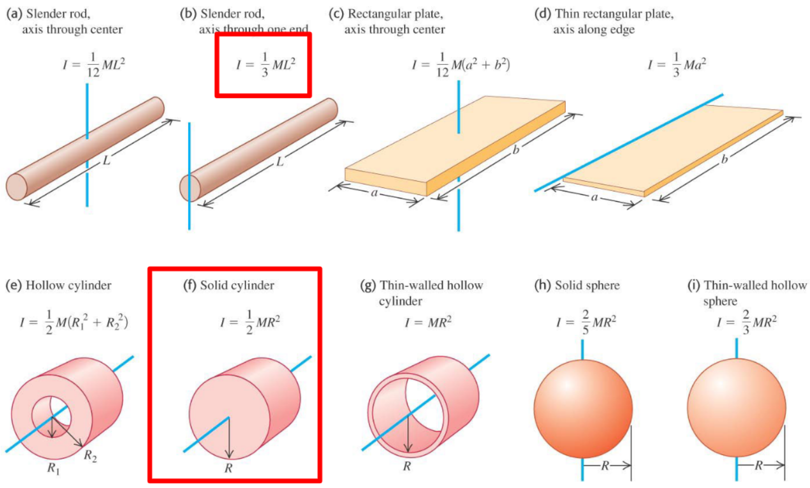
$$
\begin{aligned}
I_{CM} &= m_1\Delta x_1^2 + m_2\Delta x_2^2 \\
&= m_1(x_c - x_1)^2 + m_2(x_c - x_2)^2 \\
&= m_1x_c^2 - 2m_1x_cx_1 + m_1x_1^2 + m_2x_c^2 - 2m_2x_cx_2 + m_2x_2^2 \\
&= (m_1x_1^2 + m_2x_2^2) - 2(m_1x_1 + m_2x_2)x_c + (m_1+m_2)x_c^2 \\
&= I - 2\frac{m_1x_1 + m_2x_2}{m_1+m_2}Mx_c + Mx_c^2 \ \{M = m_1 + m_2\} \\
&= I - Mx_c^2 \ \left\{x_c = \frac{m_1x_1+m_2x_2}{m_1+m_2}\right\} \\
\Leftrightarrow I &= I_{CM} + Md^2
\end{aligned}
$$

Moments of inertia of some common bodies

- Table 9.2 gives the moments of inertia of various bodies

- Parallel axis theorem:

$$
\begin{aligned}
I_{side} &= I_{cm} + M\left(\frac{L}{2}\right)^2 \\
&= \frac{ML^2}{12} + M\left(\frac{L}{2}\right)^2 \\
&= \frac{1}{3}ML^2
\end{aligned}
$$



Disc is same

[Chapter 5B PDF]

Torque

- $\vec{\tau} = \vec{F} \times \vec{r}$, where r is the arm length
- The larger is the r, the stronger the **rotation effect**
- **line of action** of the force: along the force vector
- **level arm**: **perpendicular** distance from O (the axis of rotation) to the line of action
- When a force is exerted on the axis of rotation, **torque = 0**
- $\tau = Fl = Fr\sin\theta = F_{\perp}r$, or $\tau = Fr\sin\theta$
- direction of torque can be found using **Right Hand Rule**, [Refer to this]

Effect of Torque

- Produce acceleration in **rotational motion**, like a force can increase or decrease speed in linear motion
- $\vec{\tau}$ and $\vec{v}$ in the same direction: increase speed

Newton's Second Law for Rotation

$$\begin{aligned} \because F_t &= ma_t \\ a_t &= r\alpha \\ \therefore F_t &= mr\alpha \\ \underbrace{rF_t}_{\tau} &= \underbrace{mr^2}_{I}\alpha \\ \therefore \tau &= I\alpha \end{aligned}$$

(where: F_t is tangential external force, a_t is tangential acceleration, α is angular acceleration, I is moment of inertia)

- Don't forget: $a_{\text{rad}} = \omega^2 r$

Lots of Torques

$\Sigma\tau_i = (\Sigma I_i)\alpha = \left(\Sigma m_i r_i^2\right)\alpha = I\alpha$

Internal Forces

According to definition, action-reaction in a system will have exactly opposite torque. They will cancel each other in the total sum of torques.

$\Sigma\tau_{\text{external}} = I\alpha$

Momentum

- Momentum $\vec{p} = m\vec{v}$
- Angular momentum $\vec{L} = I\vec{\omega}$, [Refer to this]

$$\begin{gathered} \tau = I\alpha = I\frac{\Delta\omega}{\Delta t} = \frac{\Delta(I\omega)}{\Delta t} = \frac{\Delta L}{\Delta t} \\ L = I\omega = mr^2\omega = mrv = pr \\ \textbf{where } L = \textbf{ Angular momentum} \\ \Rightarrow \vec{L} = \vec{r} \times m\vec{v}\ (\vec{L} = \vec{r} \times \vec{p}) \\ (\textbf{Similiar to torque}: \vec{\tau} = \vec{r} \times \vec{F}) \end{gathered}$$

Impulse

- When a force F acts **abruptly** for a short time Δt on an object, have impulse $\vec{J} = \vec{F}\Delta t$
- Rewrite Newton's 2nd Law: $\vec{F} = m\vec{a} = m\frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta(m\vec{v})}{\Delta t}$
- in Impulse Form: $\vec{J} = \vec{F}\Delta t = \Delta(m\vec{v})$

Linear Momentum

- $E_K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$
- In a system, $\vec{P} = \Sigma(m_i\vec{v}_i^2) = \Sigma\vec{p}_i$
- **Momentum conservation** of linear momentum
  - can be found during collision
  - proof by Newton's 3rd Law: Opposite Force

- **Internal force come in action and reaction**
- When consider the whole system as one, those force momentum will be canceled
- Momentum conservation along a certain **direction** when no external force in that direction

Angular Quantities

- Momentum Conservation: $I_1\omega_1 = I_2\omega_2$
- Work: $W = F\Delta s = Fr\Delta\theta = \tau\Delta\theta$
- Power: $P = Fv = F\frac{\Delta s}{\Delta t} = Fr\frac{\Delta\theta}{\Delta t} = Fr\omega = \tau\omega$

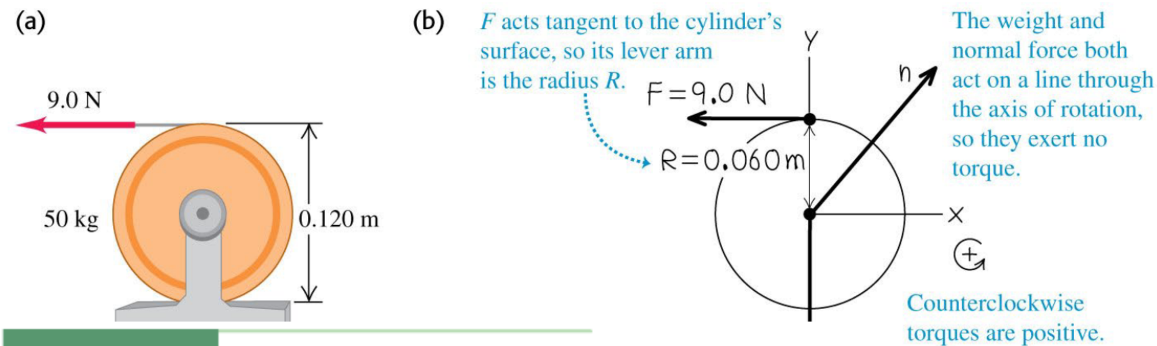
Exercise 5

Examples

Unwinding Cable I

Example 10.2 An unwinding cable I

Figure 10.9a shows the situation analyzed in Example 9.7 using energy methods. What is the cable's acceleration?



Example 9.7 An unwinding cable I

We wrap a light, nonstretching cable around a solid cylinder of mass 50 kg and diameter 0.120 m, which rotates in frictionless bearings about a stationary horizontal axis (Fig. 9.16). We pull the free end of the cable with a constant 9.0-N force for a distance of 2.0 m; it turns the cylinder as it unwinds without slipping. The cylinder is initially at rest. Find its final angular speed and the final speed of the cable.

In view of Energy

$$Fx = \frac{1}{2}Mv^2 - 0 = \frac{1}{2}I\omega^2$$

$$I = \frac{1}{2}MR^2$$

$$\therefore \omega = \sqrt{\frac{2Fx}{I}} = 20\mathrm{rad/s}$$

$$v = \omega R = 12\mathrm{m/s}$$

In view of Torque

$$\tau = FR = I\alpha$$

$$\therefore \alpha = \frac{FR}{I} = \frac{2F}{MR} = 6\mathrm{rad/s^2}$$

$$a_{\tan} = R\alpha = \frac{2F}{M} = 0.36\mathrm{m/s^2}$$

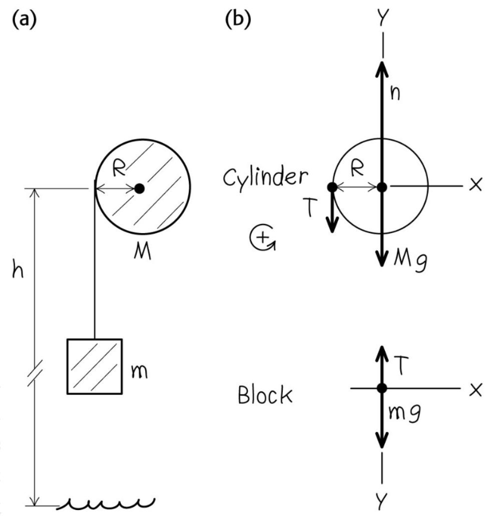
Unwinding Cable II

Example 10.3 An unwinding cable II

In Example 9.8 (Section 9.4), what are the acceleration of the falling block and the tension in the cable?

Example 9.8 An unwinding cable II

We wrap a light, nonstretching cable around a solid cylinder with mass M and radius R. The cylinder rotates with negligible friction about a stationary horizontal axis. We tie the free end of the cable to a block of mass m and release the block from rest at a distance h above the floor. As the block falls, the cable unwinds without stretching or slipping. Find expressions for the speed of the falling block and the angular speed of the cylinder as the block strikes the floor.



In view of Energy

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2$$

$$= \left(\frac{1}{2}m + \frac{1}{4}M\right)v^2$$

In view of Torque

For the cylinder, only T have a torque

$$\tau_c = RT = I\alpha = \frac{1}{2}MR^2\alpha$$

$$\Rightarrow T = \frac{1}{2}MR\alpha \qquad (1)$$

For the block, $mg - T = ma_y$ (2)

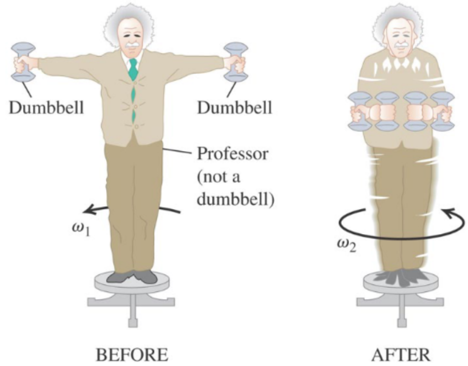
$$a_y = a_{\tan} = \alpha R \qquad (3)$$

$$\Rightarrow a = \frac{mg}{\frac{1}{2}M + m}$$

$$T = \frac{1}{2}MR\alpha = \frac{Mmg}{M + 2m}$$

Angular Momentum

A physics professor stands at the center of a frictionless turntable with arms outstretched and a 5.0-kg dumbbell in each hand (Fig. 10.29). He is set rotating about the vertical axis, making one revolution in 2.0 s. Find his final angular velocity if he pulls the dumbbells in to his stomach. His moment of inertia (without the dumbbells) is 3.0 kg · m^2 with arms outstretched and 2.2 kg · m^2 with his hands at his stomach. The dumbbells are 1.0 m from the axis initially and 0.20 m at the end.



$$I_1 = I_{\text{prof}} + I_{\text{dumb}} = I_{\text{prof}} + 2MR^2 = 3.0 + 2 \times (5\text{ kg}) \times (1\text{ m})^2 = 13.0\text{ kg} \cdot \text{m}^2$$

$$\omega_1 = \frac{1\text{ rev}}{2.0\text{s}} = 0.5\text{ rev/s}$$

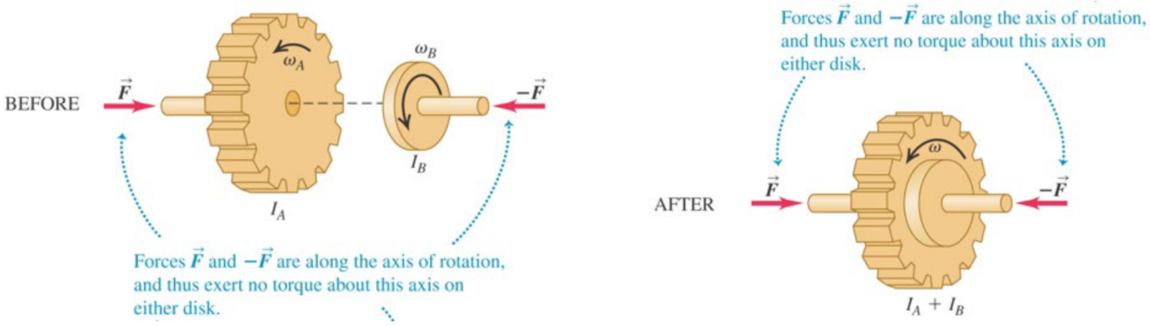
$$I_2 = 2.2 + 2 \times (5\text{ kg}) \times (0.20\text{ m})^2 = 2.6\text{ kg} \cdot \text{m}^2$$

$$I_1\omega_1 = I_2\omega_2 \Rightarrow \omega_2 = \frac{I_1}{I_2}\omega_1 = 2.5\text{ rev/s}$$

> Why the Kinetic Energy is Increased?
>
> - When the professor pulled his arm in, the dumbbell rotated in a spiral(螺旋线), **a centripetal motion** where the radius decreases.
> - So the angle between $\vec{F}$ and $\vec{v}$ is less than 90°. So E_K increased.

Example 10.11 A rotational "collision"

Figure 10.30 shows two disks: an engine flywheel (A) and a clutch plate (B) attached to a transmission shaft. Their moments of inertia are I_A and I_B; initially, they are rotating with constant angular speeds ω_A and ω_B, respectively. We push the disks together with forces acting along the axis, so as not to apply any torque on either disk. The disks rub against each other and eventually reach a common angular speed ω. Derive an expression for ω.



$$I_A\omega_A + I_B\omega_B = (I_A + I_B)\omega$$

Important: $\vec{F}$ and $-\vec{F}$ (through the rotation axis) have no torque.

Example 10.8 Calculating power from torque

An electric motor exerts a constant 10-N·m torque on a grindstone, which has a moment of inertia of 2.0 kg·m^2 about its shaft. The system starts from rest. Find the work W done by the motor in 8.0 s and the grindstone kinetic energy K at this time. What average power P_{av} is delivered by the motor?

SOLUTION

IDENTIFY and SET UP: The only torque acting is that due to the motor. Since this torque is constant, the grindstone's angular acceleration α_z is constant. We'll use Eq. (10.7) to find α_z, and then use this in the kinematics equations from Section 9.2 to calculate the angle $\Delta\theta$ through which the grindstone rotates in 8.0 s and its final angular velocity ω_z. From these we'll calculate W, K, and P_{av}.

$$\tau = I\alpha_z \Rightarrow \alpha_z = \frac{\tau}{I} = \frac{10\ \mathrm{N\cdot m}}{2.0\ \mathrm{kg\cdot m^2}} = 5\ \mathrm{rad/s^2}$$

$$\Delta\theta = \frac{1}{2}\alpha_z t^2 = \frac{1}{2}\times 5\ \mathrm{rad/s^2}\times(8\ \mathrm{s})^2 = 160\ \mathrm{rad}$$

$$\omega_z = \alpha_z t = 40\ \mathrm{rad}$$

$$\Rightarrow W = Fs = Fr\Delta\theta = \tau\theta = 1600\ \mathrm{J}$$

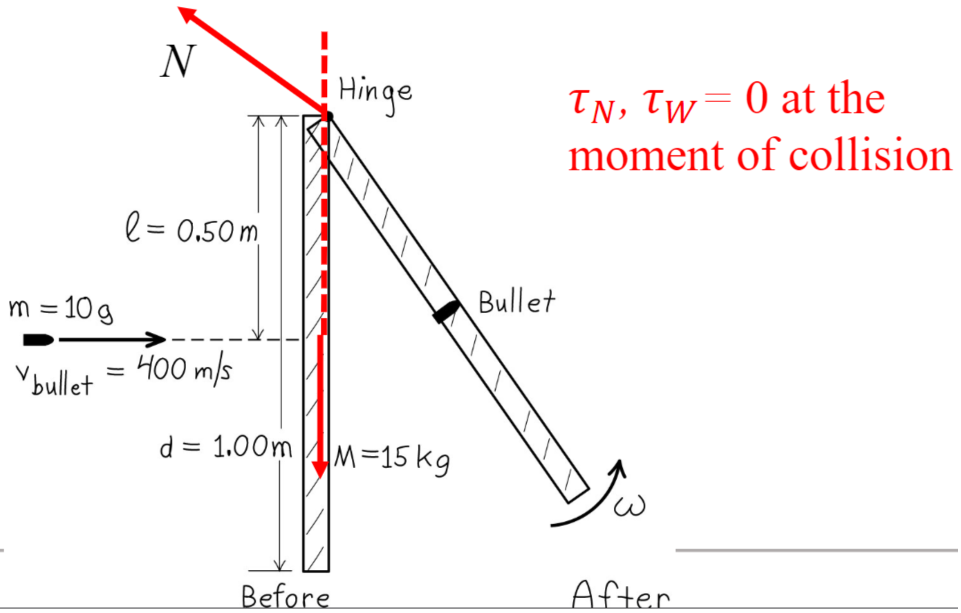
$$K = \frac{1}{2}I\omega_z^2 = 1600\ \mathrm{J}$$

$$\bar{P} = \frac{W}{t} = 200\ \mathrm{J}$$

Angular Momentum

Example 10.12 Angular momentum in a crime bust

A door 1.00 m wide, of mass 15 kg, can rotate freely about a vertical axis through its hinges. A bullet with a mass of 10 g and a speed of 400 m/s strikes the center of the door, in a direction perpendicular to the plane of the door, and embeds itself there. Find the door's angular speed. Is kinetic energy conserved?



$$\vec{L} = \vec{r}\times m\vec{v} = (0.50\ \mathrm{m})\times(0.010\ \mathrm{kg}\times 400\ \mathrm{m/s}) = 2\ \mathrm{kg\cdot m^2/s}$$

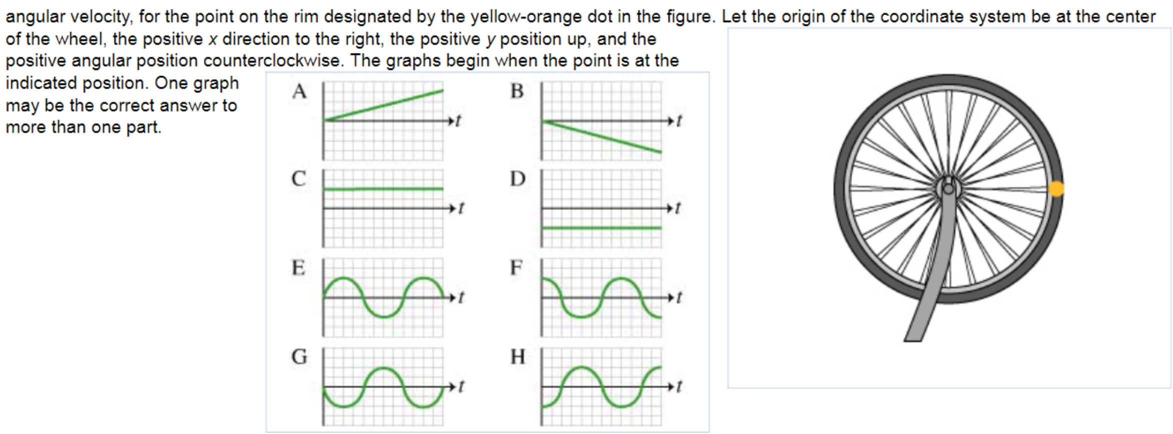
$$I_{\mathrm{door}} = \frac{1}{3}Md^2 = 5\ \mathrm{kg\cdot m^2}(\text{Rod, axis through one end, see Table 9.2})$$

$$I_{\mathrm{bullet}} = ml^2 = 0.025\ \mathrm{kg\cdot m^2}(\text{A traditional rotating object out of axis})$$

$$\omega = \frac{L}{M+m} = 0.4\ \mathrm{rad/s}$$

Tutorial

Graphs of Rotation Quantities

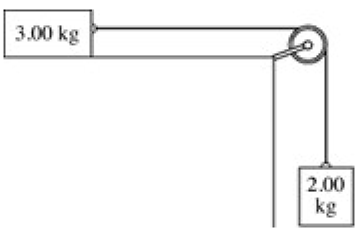


- Initially speed: Counterclockwise
- Positive Direction: x-right, y-up, θ-counterclockwise

> - $x - t$: Initially rightmost(++), going 0 => F
> - $v_x - t$: Initially 0, going left(-) => G
> - $a_x - t$: Initially leftmost(--), going 0 => H
> - $y - t$: Initially 0, going up(-) => E
> - $v_y - t$: Initially upmost(++), going 0 => F
> - $a_y - t$: Initially 0, going down(-) => G
> - $\theta - t$: Initially 0, going up(+) linear => A
> - $\omega - t$: Constant => C
> - $\alpha - t$: 0

Assignment

In the figure, two blocks, of masses $m_2 = 2$ kg and $m_1 = 3$ kg, are connected by a light string that passes over a frictionless pulley (i.e. the pulley is on a frictionless axle) of the moment of inertia $0.004\ \mathrm{kg \cdot m^2}$ and radius $5.00\ \mathrm{cm}$. However, there is sufficient friction between the pulley and the string such that the string turns the pulley without slipping (the string does not slip over the pulley). The coefficient of friction for the tabletop is $\mu = 0.2$. The blocks are released from rest. Using energy methods, find the speed of the upper block just as it has moved $3\ \mathrm{m}$.



$$
\begin{aligned}
\tau &= I\alpha \\
\Rightarrow T_2R - T_1R &= I\frac{a}{R} \\
m_2g - T_2 &= m_2a \\
\Rightarrow T_2 &= m_2g - m_2a \\
T_1 - \mu m_1 g &= m_1 a \\
\Rightarrow T_1 &= \mu m_1 g + m_1 a \\
\therefore (m_2g - m_2a - \mu m_1 g - m_1 a) &= I\frac{a}{R}
\end{aligned}
$$

Ch6 Temperature and Heat

[Chapter 6 PDF]

Heat

- A form of **energy**, unit=J
- Receive Energy = Temperature Up
- Heat can be **converted into work**

Temperature

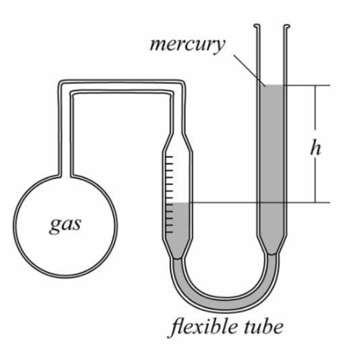
- The degree of Hotness
- measured using **Thermometer**, temperature can be calculated in a linear way
  $l = a + b \times T$
- **Gas Thermometer**

The pressure of a gas with fixed volume is used to measure the temperature. The pressure of the gas is measured by the length of the mercury column, h.

Temperature increases (hotter), pressure increases. h increases. Pressure tells you the temperature

Relation between pressure and temperature (C)
$P = a + bT_C$

From pressure you can find T_C



Pressure of gas=P_1, Pressure outside=P_2
$P_1 = P_2 + \rho gh$; $\rho = density\ of\ mercury$,
$g = gravitational\ acceleration$,
$h = difference\ between\ the\ two\ levels$

Temperature Scale

- Celsius $P = a + bT_C$
- Fahrenheit
- Kelvin $P = bT_K$, $\frac{T_1}{T_2} = \frac{P_1}{P_2}$
- $P = a + bT_C = T_K \Rightarrow T_C = T_K - d, a = bd$
- **Triple Point of water is 273.16K**, so $T = 273.16\frac{P}{P_{\text{triple}}}$
- **Boiling Point of water is 373.15K**

Temperature Scale Conversions

- $T_F = \frac{9}{5}T_C + 32^\circ$
- $T_C = \frac{5}{9}(T_F - 32^\circ)$
- $T_K = T_C + 273.15$

Thermal Equilibrium

- When two objects have different **temperature**, there is **heat flow** between the two objects
- When they have same temperature, they are in **thermal equilibrium**, there is no heat flow
- In Temperature Equilibrium: **The thermometer and the object are in thermal equilibrium**

0th Law of Thermodynamics

- C and A in equilibrium, C and B in equilibrium $\Rightarrow$ A and B in equilibrium
- **Two objects are in equilibrium if they have same temperature**

Heat is Energy

- Heat energy come from **work** (done into the object) and **potential energy**
- Heat is a form of energy
- **Internal Energy = Random E_K + Inter-molecular E_P**
- Temperature Increase = Internal Energy Increase = **Heat** energy consumed

Thermal State

Change in Temperature > Change in Thermal State > Change in Properties

State Variables

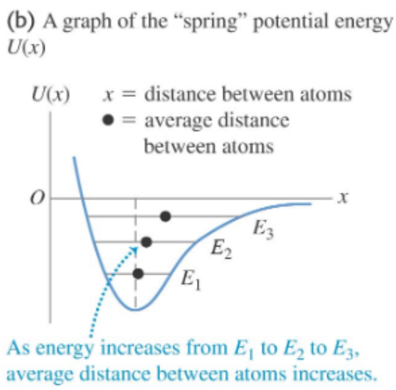
- Temperature, Pressure, Volume (Length)
- Describes thermal state of matter

Thermal Expansion

- Linear Expansion $\Delta L = \alpha L_0 \Delta T$
- Hole Expansion $\Delta A = \mu A_0 \Delta T, \mu = 2\alpha$, does not shrink
- Volume Expansion $\Delta V = \beta V_0 \Delta T, \beta = 3\alpha$
- Liquid only have β

Atomic Mechanism

- atoms are in **constant random motion**, bound together by interatomic forces
- Higher Temperature, larger interatomic E_P
- **Repulsive Force > Attractive Force, so they pull apart**
- **Average Position** between atoms increase



Water

- Between $0^{\circ}\mathrm{C}$ and $4^{\circ}\mathrm{C}$ water decreases in volume with increasing temperature
- Hydrogen Bond between molecules is strong
- Example: **Lakes freeze from the top down**

Thermal Stress

- Stress $\frac{F}{A} = -Y\alpha\Delta T$
- Strain = $\alpha\Delta T$
- Example: Gaps between Bridges can protect it from bending

Stress

- Length is shortened: Compressive
- Length is lengthened: Tensile
- **Stress =** $\frac{F_{\perp}}{A}$
- **Strain =** $\frac{\Delta l}{l_0}$
- $Y = \frac{\text{Tensile Stress}}{\text{Tensile Strain}} = \frac{F_{\perp} l_0}{A\Delta l}$

Specific Heat

- $Q = mc\Delta T$
- $1\ \mathrm{J} \cdot \mathrm{K}^{-1}\mathrm{g}^{-1} = 10^3\ \mathrm{J} \cdot \mathrm{K}^{-1}\mathrm{kg}^{-1}$

Phase Changes

- **Temperature does not change**
- Heat of Fusion L_f
- Heat of vaporization L_v
- Heat transferred $Q = \pm mL$

Heat Transfer

Conduction of Heat (Heat Current)

- $H = \frac{\Delta Q}{\Delta t} = \frac{kA(T_H - T_C)}{L}$
  - A is section area
  - L is conductor length
  - T_H, T_C is hot-end and cold-end temperature
  - k is thermal conductivity
  - **With multiple conductors**, sum up the k
- Mechanism: **Rapid moving atoms cause neighboring atoms to move fast**

Convection of Heat

- Through Fluid Motion
- Temperature increases, **liquid/gas expand, density decreased, hotter fluid move up** and move to the cold area

Radiation of Heat

- Non-zero temperature
- Low-temp: Infra Red
- High-temp: Visible & Invisible
- $H = Ae\sigma(T^4 - T_s^4)$
  - A is surface area
  - e is emissivity
  - $\sigma = 5.67 \times 10^{-8}\ \mathrm{W/m^2K^4}$
  - T, T_s is surface and surrounding temperature

Training 6

Heat Conductivity

(Assignment 6 Question 4)

The wall of a house is made from a layer of wood $3\ \mathrm{cm}$ thick and a layer of wool which is $5.5\ \mathrm{cm}$ thick. The layer of wood faces outside and the layer of wool faces inside. The thermal conductivity of wood is $0.08 \mathrm{W/mK}$ and the thermal conductivity of wool is $0.02 \mathrm{W/mK}$. The temperature outside the house is 0° and the temperature inside the house is 21.8°. What is the temperature at the contact surface between the wool and the wooden layers?

$\frac{K_{\text{out}}A}{l_{\text{out}}}(T - T_{\text{out}}) = \frac{K_{\text{in}}A}{l_{\text{in}}}(T_{\text{in}} - T)$

`out` is wood, `in` is wool

Ch7 Thermal Properties of Matter

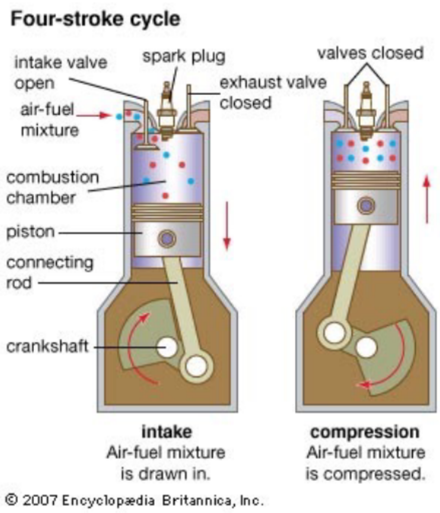
[Chapter 7 PDF]

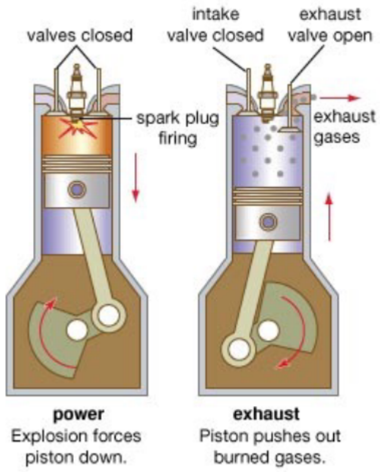
Thermal State of Matter

State Variables

- Pressure, volume, temperature
- move heat from hot to cold **using change of thermal states**, e.g. refrigerators

4-Stroke Engine





	Temperature	Pressure	Volume
Intake	-	-	+
Compression	+	+	-
Power	+	+	+
Exhaust	-	-	-

In *Intake* and *Exhaust*, hot air go to outside and suck in cold air-fuel mixture.

Equation of State

- $pV = nRT, R = 8.31\ \mathrm{J} \cdot \mathrm{K}^{-1}\mathrm{mol}^{-1}$

> - Pressure ∝ number of molecule: $P \propto n$
> - Pressure = force / area: $P \propto L^{-2}$
> - Pressure ∝ number of collision $\left(= \frac{V}{2L}\right)$: $P \propto L^{-1}$
> - $P \propto L^{-2}$ and $P \propto L^{-1} \Rightarrow P \propto \frac{1}{V} \Rightarrow P \propto \frac{n}{V}$

Ideal gas

- Equation Only Ideal gas Behavior
- Assumption1: Molecule Volume is 0
- Assumption2: No Interactions between Molecules
- At normal temperature and pressure, all gases have behavior resembling the ideal gas
- **At high temperature / low pressure**, all gases behaves like the ideal gas
- **Standard Temperature and Pressure** (STP): $$T = 0^\circ \text{ C} = 273.15 \text{ K}$$ $$p = 1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$
- At STP, 1 mole of an ideal gas occupies $V = \frac{nRT}{p} = 22.4 \text{ L}$, notice $1 \text{ L} = 1 \times 10^{-3} \text{ m}^3$

Interference

- $m = nM$, where M is molecular weight
- $\rho = \frac{m}{V} = \frac{nM}{V} = \frac{pM}{RT}$
- $\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$
- $M = 28.8 \text{ g/mol} = 28.8 \times 10^{-3} \text{ kg/mol}$

Number of Moles

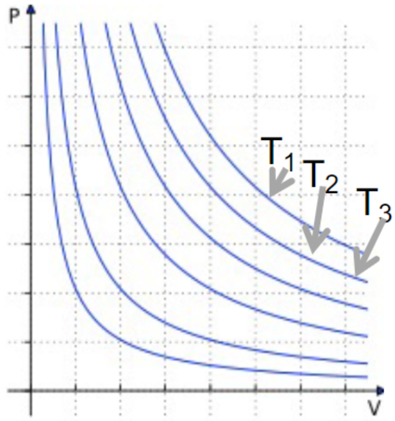
- $1 \text{ mol } ^{12}C = 0.012 \text{ kg}$
- $1 \text{ mol } ^{12}C = 6.022 \times 10^{23} \text{ of } ^{12}C$
- $M = N_A m$, M is one mole weight, m is one molecule weight

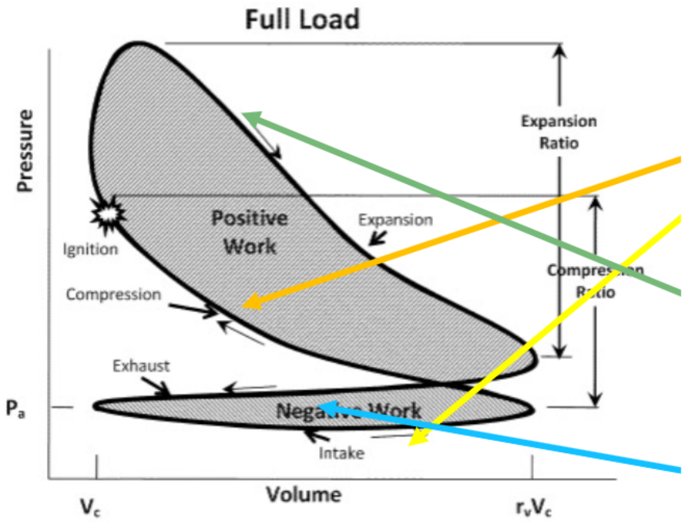
The van der Waals equation

- $\left(p + \frac{an^2}{V^2}\right)(V - nb) = nRT$
- $+\frac{an^2}{V^2}$
  - **Inter-molecule Attractive Force**
  - Actual pressure > measured pressure $= p$
  - two molecules have interaction, so $\propto n^2$
- $-nb$
  - volume of molecules
  - Actual total volume < measured volume $= V$

$pV-$diagrams

- $p \propto \frac{1}{V}$
- $pV \propto T$
- $T_1 > T_2 > T_3$ in the following graph





$pT-$diagrams

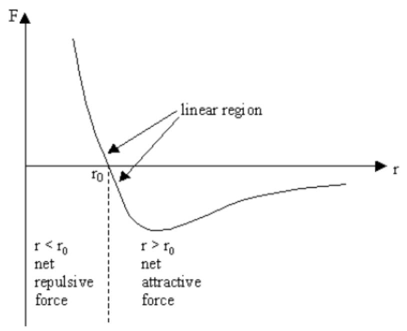
- straight lines, $p \propto T$, all lines intersects at $(0\ \mathrm{K}, 0)$
- $k = \frac{p}{T} \propto \frac{1}{V}$

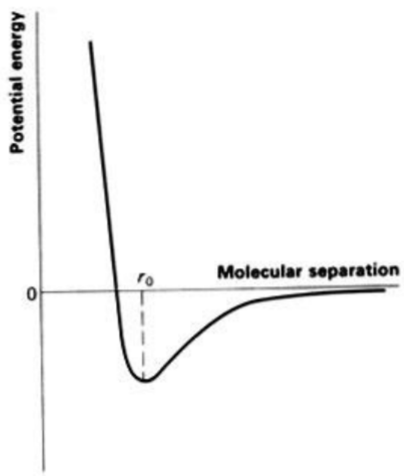
$VT-$diagrams

- straight lines, $V \propto T$, all lines intersects at $(0\ \mathrm{K}, 0)$
- $k = \frac{V}{T} \propto \frac{1}{p}$

Microscopic Models

Forces between Molecules





(UP: force; DOWN: Potential Energy)

- distance between molecules: gas > liquid > solids

- $F_r(r) = -\frac{dU}{dr}$, U is potential energy

Kinetic-Molecular Model

Assumption

- The molecules size much smaller than the **average distance between molecules**
- The molecules undergo **perfectly elastic collision**

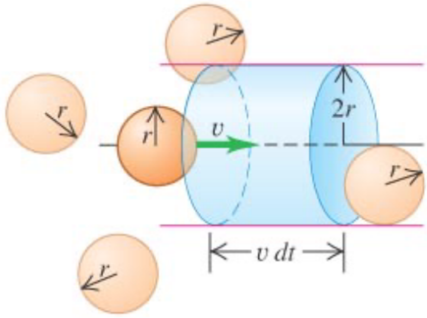
Collision With Wall

- With speed $v = \{-|v_x|, v_y\}$, when colliding with the wall on $x-$direction, the speed
  - $x-$component becomes $|v_x|$.
  - $y-$component stays the same.
  - magnitude stays the same.
- **Momentum change** $\Delta p = (m|v_x|) - (-m|v_x|) = 2m|v_x|$
- **During short time** dt,
  - the molecules traveled on $x-$component at an average distance of $|v_x|dt$.
  - on a certain area A, the number of molecules hitting the wall is $\frac{1}{2}\frac{N}{V}(A|v_x|dt)$
  - where N is the number of molecules in total volume V
  - and the total momentum change $dp_x = \frac{1}{2}\frac{N}{V}(A|v_x|dt) \times \Delta p = \frac{NAmv_x^2 dt}{V}$
- **Pressure = Force / Unit Area** $p = \frac{F}{A} = \frac{1}{A}\frac{dp_x}{dt} = \frac{Nmv_x^2}{V} \Leftrightarrow {\color{red} pV = Nm < v_x^2 >}$
  - where $< v_x^2 >= \bar{v_x^2}$, the average of square (in kinetic way)
- Because $< v_x^2 >=< v_y^2 >=< v_z^2 >= \frac{1}{3} < v^2 >$,
  - $pV = \frac{1}{3}Nm < v^2 >= \frac{2}{3}N\frac{1}{2}m < v^2 >= \frac{2}{3}N \cdot E_k = \frac{2}{3}K_{\text{tr}}$
  - where K_{tr} is total translational Kinetic Energy
- According to $pV = nRT$, we have internal energy $U = {\color{red} K_{\text{tr}} = \frac{3}{2}nRT = \frac{3}{2}Nk_BT}$
- **Heat Capacity** per mole $C_V = \frac{\Delta U}{\Delta T} = \frac{3}{2}N_Ak_B$
- **Average translational Kinetic Energy** of a molecule $\frac{K_{\text{tr}}}{N} = \frac{3}{2}\frac{n}{N}RT = \frac{3}{2}\frac{R}{N_A}T$
  - where n is total mole of gas, N is total molecule of gas, so $N = N_An$
  - **Boltzmann constant** ${\color{blue} k_B = \frac{R}{N_A} = 1.381 \times 10^{-23}}$
  - $\frac{1}{2}m < v^2 >= \frac{3}{2}k_BT \Leftrightarrow {\color{red} v_{\text{rms}}} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3RT}{mN_a}} = {\color{red} \sqrt{\frac{3RT}{M}}}$
  - $v_{\text{rms}} = \sqrt{\frac{v_1^2+v_2^2+\cdots+v_n^2}{n}} > \frac{v_1+v_2+\cdots+v_n}{n} = v_{\text{av}}$
  - `rms` stands for `root-mean-square`
  - Beware that $M = N_Am$ **results in** g **, remember to convert to** kg
  - $K_B = \frac{R}{N_A}, N_A = \frac{N}{n} \Rightarrow {\color{red} pV = Nk_BT} \Rightarrow pV = \frac{2}{3}U$

Collisions between Molecules

- **mean free path**: average distance between collisions
- **mean free time**: average time between collisions

Collisions between molecules



- During dt, a molecule can collide with any molecules within the cylinder of radius 2r and length vdt

$$dN = 4\pi r^2 v\, dt\, N/V$$

- Number of collisions per unit time (factor sqrt(2) is due to the actual collisions are more frequent)

$$\frac{dN}{dt} = \frac{4\pi\sqrt{2}r^2 vN}{V}$$

$$t_{mean} = \frac{V}{4\pi\sqrt{2}r^2 vN} \quad \text{(mean free time of a gas molecule) (18.20)}$$

$$\lambda = \frac{V}{4\pi\sqrt{2}r^2 N} \quad \text{(mean free path of a gas molecule) (18.21)}$$

$$\lambda = \frac{kT}{4\pi\sqrt{2}r^2 p} \quad \text{(by using PV=NkT) (18.22)}$$

- $t_{\text{mean}} = \frac{\lambda}{v_{\text{rms}}} = \frac{kT}{4\sqrt{2}\pi r^2 p} \div \sqrt{\frac{3RT}{M}}$

Principle of Equipartition of Energy

- $E_K = \frac{3}{2}kT \Rightarrow E_{kx} = E_{ky} = E_{kz} = \frac{1}{2}kT$
- also applies to **rotational degree of freedom**

Heat Capacity

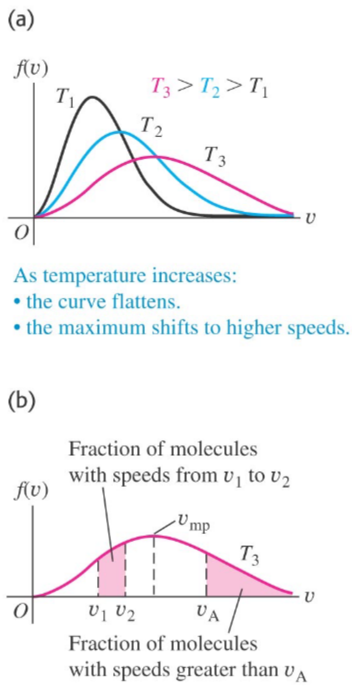
- Specific Heat $c = \frac{C}{m}$

> - $dK_{\text{tr}} = \frac{3}{2}nRdT$
> - $dQ = mcdT = nMcdT = nCdT$

- $C_V = \frac{3}{2}R$ (for **monatomic** ideal gas 单原子)
- $C_V = \frac{5}{2}R$ (for diatomic ideal gas 双原子)

Molecular Speed

- **Speed Distribution** of Gas Molecules
- **Area Under the curve** is the fraction of molecules with velocity in that range



Phases of Matter

- **Boiling Point Depends on Pressure**

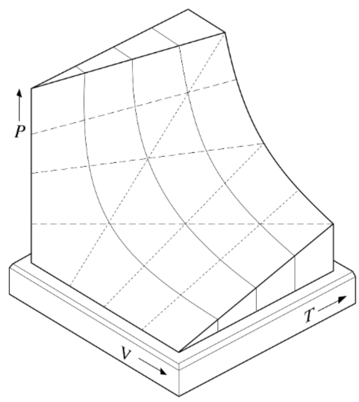
$pVT-$Diagram

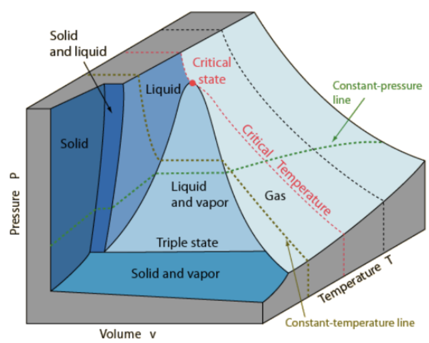
Each point in the PVT surface gives the values of P V and T for a fixed amount of gas.

Each point represents a state of the gas.

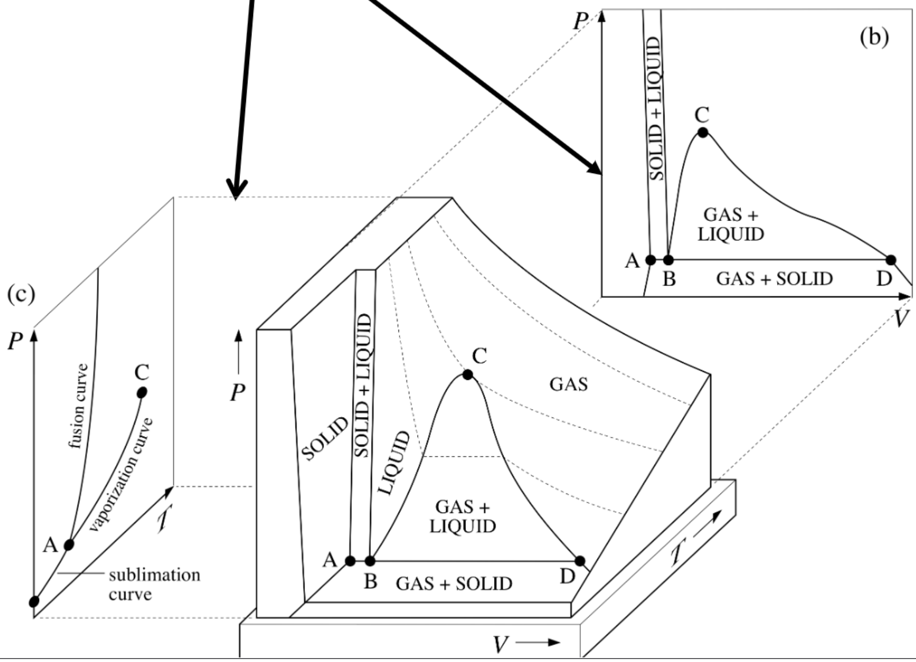
For ideal gas, no liquid or solid phase, so PTV surface is smooth.

For real gas, owing to the attractive force between atoms or molecules, the solid and liquid phases are formed, the PVT surface is distorted, not smooth.





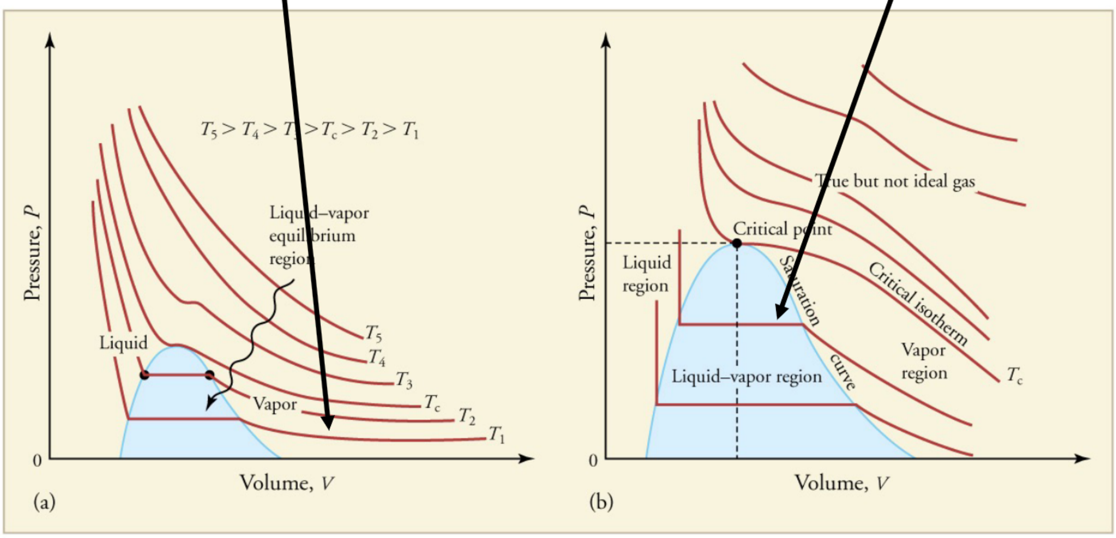
PT and PV phase diagram can be considered as the projection of the PVT surface on the PT and PV plane



$pV-$**Diagram**

When pressure changes volume changes, the vapor is compressed or expanded

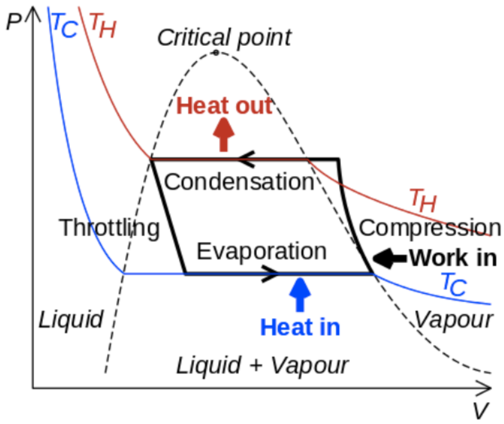
Liquid-vapor coexist here. When volume increases, liquid change to vapor Volume decrease vapor changes to liquid

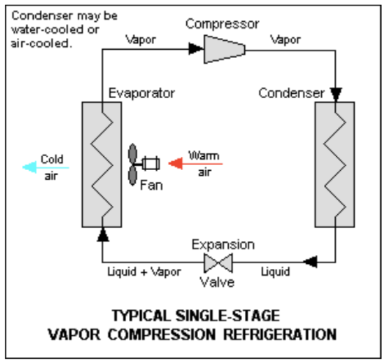


Refrigerator

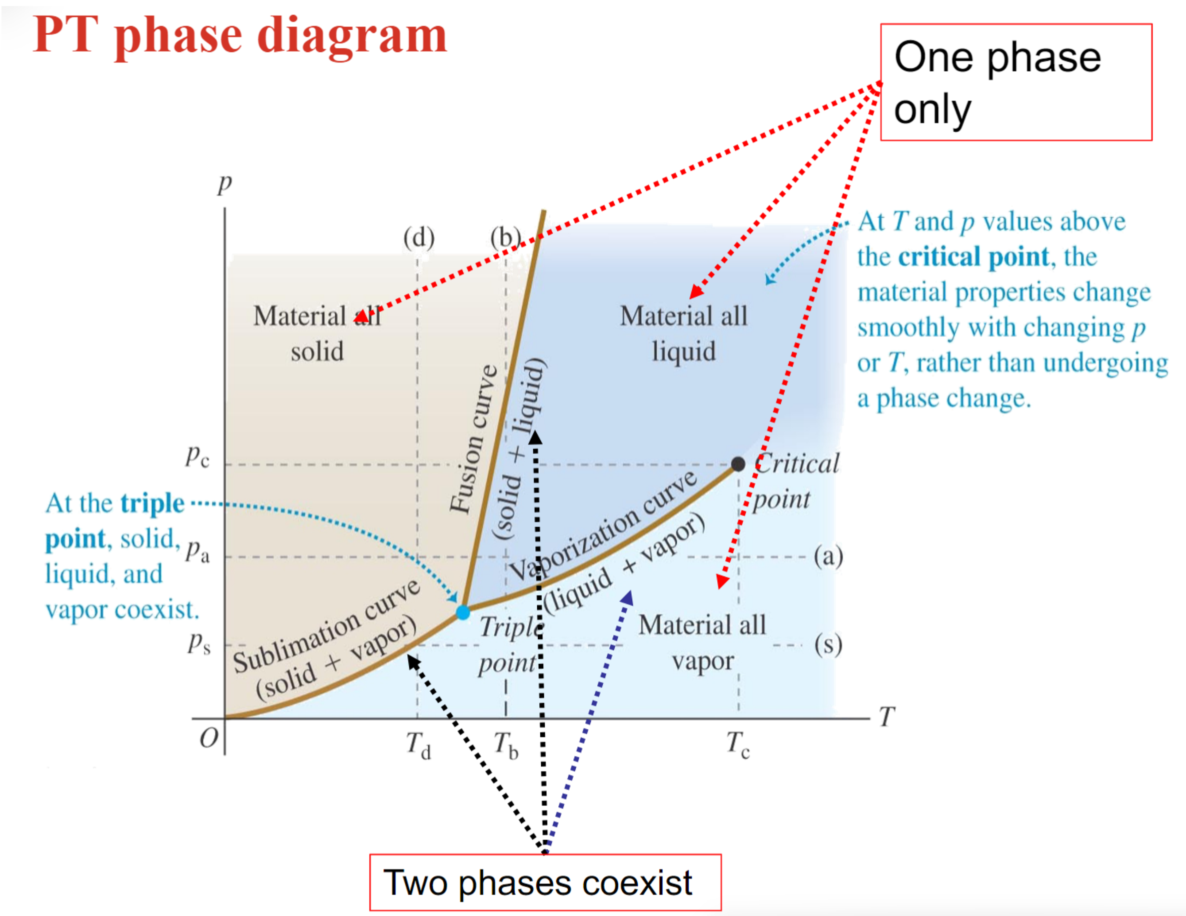
4 steps:

- Condensation: vapor changes to liquid, give out heat (feel warm at back of refrigerator)
- Throttling: expansion through a valve, liquid becomes liquid+vapor
- Evaporation: liquid changes to vapor, absorb heat (cool the food inside)
- Compression: vapor compressed, vapor volume is decreased, need work done in this step





$pT-$Diagram



- **Critical Point**:
  - Below Critical Point, there are liquid + vapor
  - **Right-above** Critical Point, there are only **supercritical fluid**
- **Triple Point**: Three phases coexist

Training 7

Pressure Change Rate

(Example 18.4)

Find the variation of atmospheric pressure with elevation in the earth's atmosphere. Assume that at all elevations, $T = 0^\circ\mathrm{C}$ and $g = 9.80\ \mathrm{m/s^2}$.

$$
\begin{aligned}
\because \frac{dp}{dy} &= -\rho g \\
\rho &= \frac{pM}{RT} \\
\therefore \frac{dp}{dy} &= -\frac{pMg}{RT} \\
\int_{p_1}^{p_2} \frac{dp}{p} &= -\frac{Mg}{RT}\int_{y_1}^{y_2} dy \\
\ln \frac{p_2}{p_1} &= -\frac{Mg}{RT}(y_2 - y_1) \\
\therefore \frac{p_2}{p_1} &= e^{-Mg(y_2-y_1)/RT} \\
\therefore p &= p_0 e^{-Mgy/RT}
\end{aligned}
$$

where $p_0 = 1.013 \times 10^5\ \mathrm{Pa}$, $y_1 = 0$ at sea level.

At $y = 8863\ \mathrm{m}$, $T = 0^\circ\mathrm{C}$, $p = p_0 \exp\frac{-28.8\times10^{-3}\mathrm{kg/mol}\times9.8\mathrm{m/s^2}\times8863\mathrm{m}}{8.314\mathrm{J/mol{\cdot}K}\times273\mathrm{K}} = 0.33\ \mathrm{atm}$

Ch8 Mechanical Waves

[Chapter 8 PDF]

Definition

- A pattern travelling through the medium, **involving the motion of particles of the medium**

Types

- Transverse: motion of particle **perpendicular to wave propagation** 横波
- Longitudinal: motion of particle parallel to wave propagation 纵波
- Periodicity: take the same amount of **time** to repeat
- Oscillation: move back and forth between **two states**

Periodic Transverse Waves

- The particles **only move up and down**, not move to left or right
- **Crest, Trough** 波峰，波谷
- $A =$**Amplitude** 振幅
- $\lambda =$Wavelength $Tv = \lambda$
- $T =$Period, $f =$frequency
- $v =$speed of the wave, $v = f\lambda$

- **Sinusoidal** means the pattern is described by $\sin$ or $\cos$ functions
- $y(x,t) = A\cos(kx-\omega t) = A\cos\left(k\left(x-\frac{\omega}{k}t\right)\right) \Rightarrow v = \frac{\omega}{k}$ (moving +x)
- $y(x,t) = A\cos(kx+\omega t)$ (moving -x)
- $y(x,t)$ is a function of x for fixed t; a function of t for fixed x

Periodic Longitudinal Waves

- **Compression, Rarefaction** 密部，疏部
- λ, T, f, v
- Compression & Rarefaction moves with the wave
- $\Delta x(x,t) = A\cos(kx-\omega t)$(moving +x)
- $\Delta x(x,t) = A\cos(kx+\omega t)$ (moving -x)
- Δx is the displacement from equilibrium position

Calculation

$\omega = \frac{2\pi}{T} = 2\pi f$ Angular Frequency

$f\lambda = v$

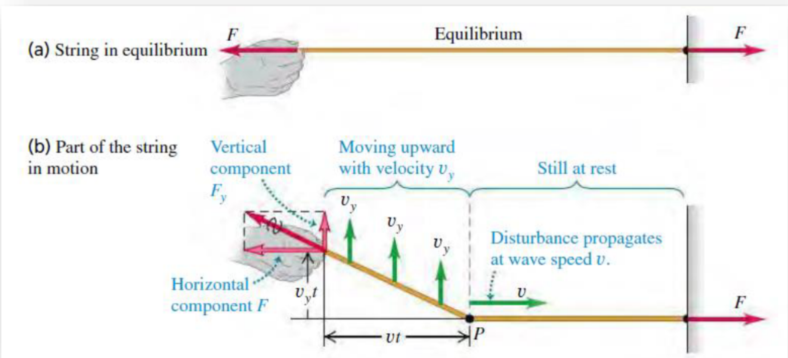
$k = \frac{2\pi}{\lambda}$ Wave Number

$\Rightarrow \omega = vk \Leftrightarrow v = \frac{\omega}{k}$

The speed of a transverse wave on a string

Tension in string : F

Linear mass density: μ



$$\frac{F_y}{F} = \frac{v_y t}{vt}$$

$$\text{Transverse impulse} = \text{Transverse momentum}$$

$$F_y t = m v_y$$

$$F_y t = (\mu v t) v_y \quad \Rightarrow F\frac{v_y}{v} = \mu v v_y \text{ or } v^2 = \frac{F}{\mu}$$

$$\boxed{v = \sqrt{\frac{F}{\mu}} \qquad (15.13)}$$

- notice **it is a stretched string** so the F_y must be along the direction of string
- $F(\mathrm{N}), \mu(\mathrm{kg/m})$

Power of energy transfer in a wave

For a sinusoidal wave

$$y(x,t) = A\cos(kx - \omega t)$$

$$\frac{\partial y(x,t)}{\partial x} = -kA\sin(kx - \omega t)$$

$$\frac{\partial y(x,t)}{\partial t} = \omega A\sin(kx - \omega t)$$

The instantaneous power $P(x,t) = Fk\omega A^2 \sin^2(kx - \omega t)$

$$P(x,t) = \sqrt{\mu F}\,\omega^2 A^2 \sin^2\left[\omega\left(\frac{x}{v} - t\right)\right] \qquad (15.23)$$

$v = \frac{\omega}{k} = \sqrt{\frac{F}{\mu}}$

The maximum value

$$\boxed{P_{\max} = \sqrt{\mu F}\,\omega^2 A^2 \qquad (15.24)}$$

The *average* power P_{av}

$$\boxed{P_{av} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2 \qquad (15.25)}$$

Average means average over a cycle

- $P_{\mathrm{av}} = \frac{1}{2} P_{\max}$
- P is max at $|\sin(kx - \omega t)| = 1 \Rightarrow \cos(kx - \omega t) = 0$, at equilibrium
- P is min at $\sin(kx - \omega t) = 0 \Rightarrow |\cos(kx - \omega t)| = 1$, at top

Wave Intensity

- **Average power carries per unit area**, in a 3D spherical space
- **Intensity = Total Power/**$4\pi r^2$
- $I \propto 1/r^2 \Rightarrow \frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$

Reflected Wave

Fixed End

- Opposite Force
- Same Amplitude, **Reversed Direction (+x -> -x)**

Free End

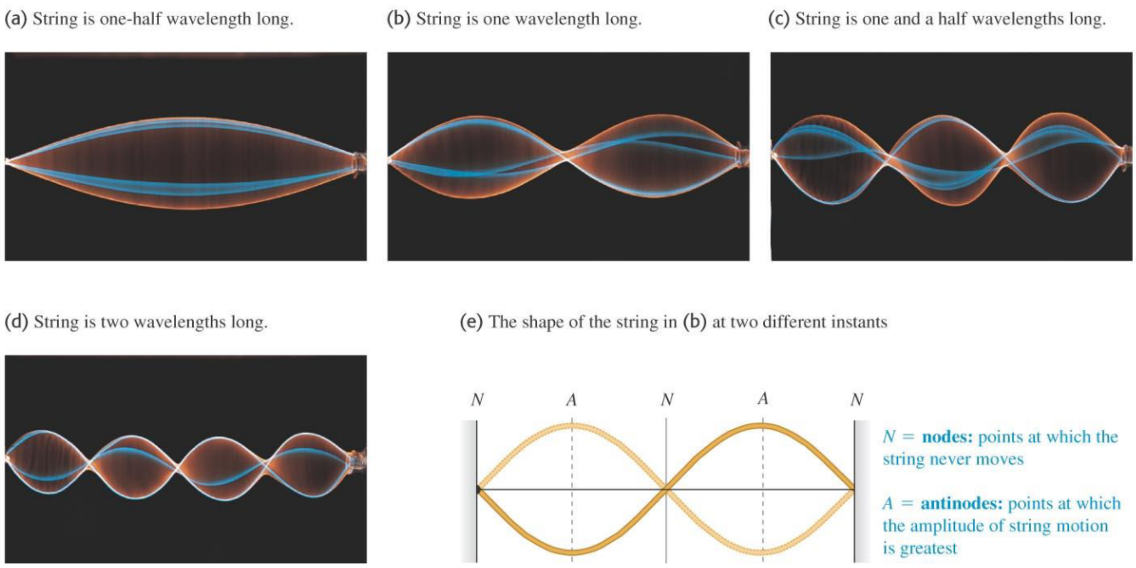
- Same Direction Force
- **Same Direction (+x -> +x)**

Wave Interference

Superposition

$y(x,t) = y_1(x,t) + y_2(x,t)$

Standing Wave

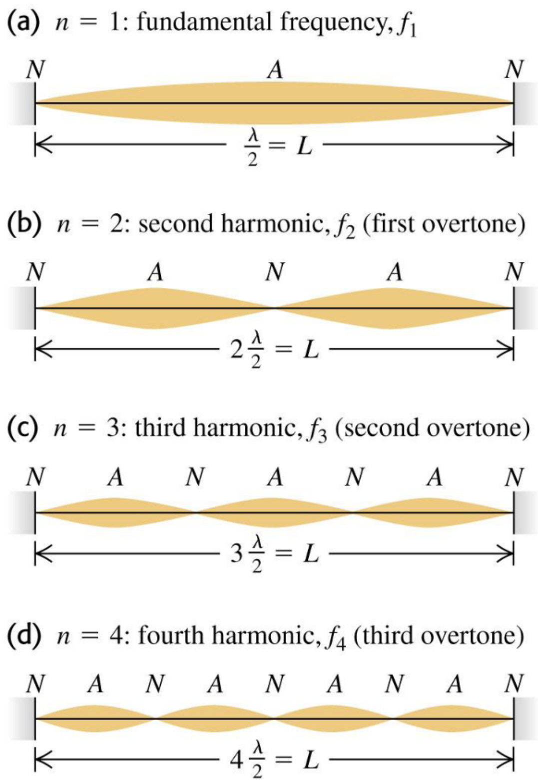


- Standing Wave: Two waves, **same A, same f, opposite direction**
- It looks no left/right move. The pattern only move up/down
- **Nodes**(N): 0 Motion, **Destructive Interference** (Displacement Cancel each other)
- **Antinodes**(A): Greatest Motion, **Constructive Interference** (Displacement Add)
- $y_1(x,t) = -A\cos(kx+\omega t)$(moving -x)
- $y_2(x,t) = A\cos(kx-\omega t)$(moving +x)
- $y(x,t) = y_1(x,t) + y_2(x,t) = (2A\sin kx)\sin\omega t$
- **New amplitude = 2A**
- Nodes at $x = 0, \frac{\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2}, \cdots \frac{n\pi}{k} = \frac{n\lambda}{2}$

$$\begin{aligned}
y(x,t) &= y_1(x,t) + y_2(x,t) \\
&= -A\cos kx\cos\omega t + A\sin kx\sin\omega t + A\cos kx\cos\omega t + A\sin kx\sin\omega t \\
&= 2A\sin kx\sin\omega t \\
v(x,t) &= \frac{\Delta y(x,t)}{\Delta t} \\
&= 2A\omega\sin kx\cos\omega t \\
a(x,t) &= \frac{\Delta a(x,t)}{\Delta t} \\
&= -2A\omega^2\sin kx\sin\omega t = -2\omega^2 y(x,t)
\end{aligned}$$

Normal modes of a string

- For a taut string fixed at both ends, the possible wavelengths are $\lambda_n = 2L/n$ and the possible frequencies are $f_n = n\,v/2L = nf_1$, where $n = 1, 2, 3, \ldots$
- f_1 is the *fundamental frequency*, f_2 is the second harmonic (first overtone), f_3 is the third harmonic (second overtone), etc.
- Figure at right illustrates the first four harmonics.



notice: **string fixed at both ends**

- $\lambda_n = \frac{2L}{n}$
- $f_n = n\frac{v}{2L} = nf_1$
- $f_1 = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$
- Longer string, lower frequency sound (e.g. String Length: Bass viol > Cello > Viola > Violin)

Tutorial

Waves on Strings (P. 4)

- In summary, when a wave travels along a string, the wave speed depends exclusively on the properties of the string, whereas the **wave frequency is set by the oscillator** that creates the waves. The wavelength is a quantity that can vary if either the wave speed or the wave frequency is changed. Thus, it can be modified by changing either the motion of the oscillator or the properties of the string.

Normal Mode and Resonance Frequencies (P. 6)

A *normal mode* of a closed system is an oscillation of the system in which all parts oscillate at a single frequency. In general there are an infinite number of such modes, each one with a distinctive frequency f_i and associated pattern of oscillation.

Consider an example of a system with normal modes: a string of length L held fixed at both ends, located at $x = 0$ and $x = L$. Assume that waves on this string propagate with speed v. The string extends in the *x* direction, and the waves are transverse with displacement along the *y* direction.

In this problem, you will investigate the shape of the normal modes and then their frequency.

The normal modes of this system are products of trigonometric functions. (For linear systems, the time dependance of a normal mode is always sinusoidal, but the spatial dependence need not be.) Specifically, for this system a normal mode is described by

$$y_i(x,t) = A_i \sin\left(2\pi\frac{x}{\lambda_i}\right)\sin\left(2\pi f_i t\right).$$

- λ_i satisfying **y(0, t)=y(L, t)=0**, corresponding to two nodes
- $\lambda_i = \frac{2L}{i}, f_i = \frac{v}{\lambda_i} = i\frac{v}{2L}$

Ch9 Sound Wave

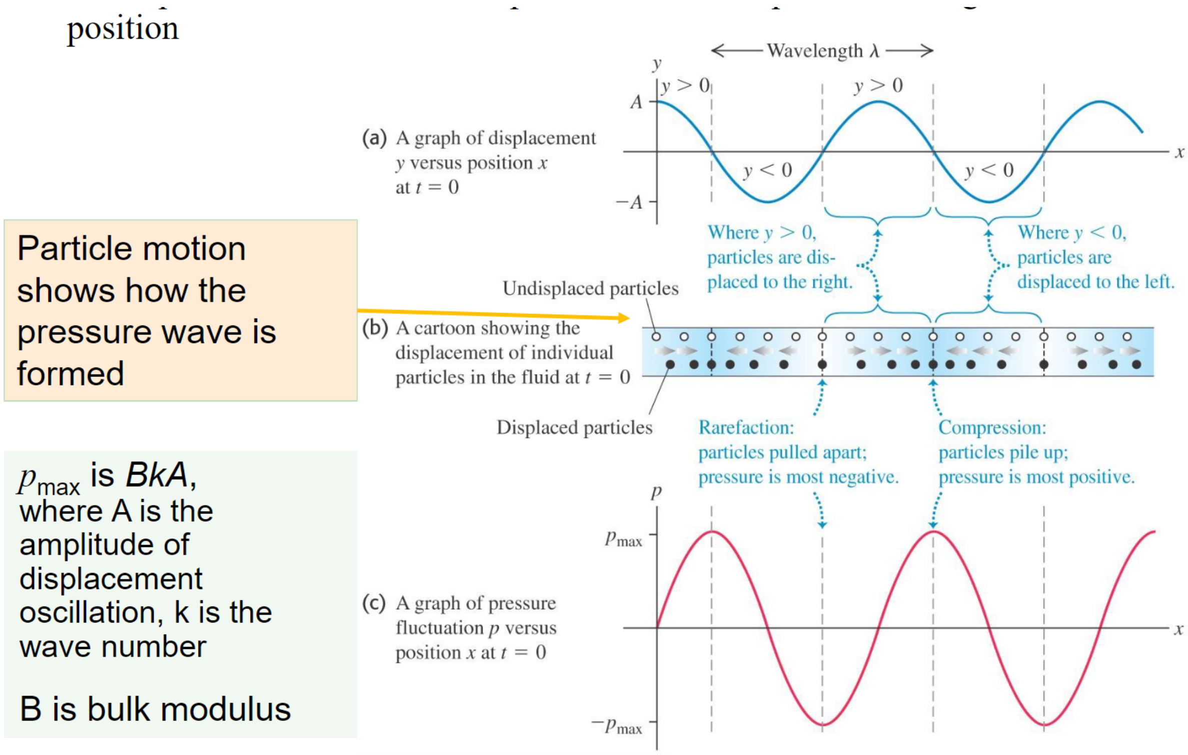
[Chapter 9 PDF]

Bulk Modulus

- Sound Wave is also **Pressure Wave**
- Bulk Modulus $\Delta p = -B\frac{\Delta V}{V_0}$
  - $B = \frac{\text{Bulk Stress}}{\text{Bulk Strain}} = -\frac{\Delta p}{\Delta V/V_0}$
  - Object under **bulk stress**, $p = p_0 + \Delta p$ increases, $V = V_0 + \Delta V(\Delta V < 0)$ decreases

Pressure Fluctuations

- $y(x,t) = A\cos(kx - \omega t)$
- $\frac{\Delta V}{V_0} = \frac{(y_2-y_1)S}{(x_2-x_1)S} = \frac{y(x_1+\Delta x)-y(x_1)}{\Delta x} = y'|_{x=x_1} = -Ak\sin(kx_1 - \omega t)$
- $p(x,t) = -B\frac{\Delta V}{V_0} = BkA\sin(kx - \omega t)$
- $p_{\max} = BkA$
- $v = \frac{\omega}{k} \Rightarrow k = \frac{\omega}{v}$



- Rarefaction 疏部: - pressure, + volume
- Compression 密部: + pressure, - volume
- **Displacement = Cosine, Pressure = Sine**
- **Pressure = 0, Displacement = Max**
- **Displacement = 0, Pressure = Max**

Speed of Sound Wave

- Music contains sounds of different frequencies (**harmonic content**)
- **Fluid** $v = \sqrt{\frac{B}{\rho}}$ Bulk modulus/density
- **Ideal Gas** $\sqrt{\frac{\gamma RT}{M}}$, $\gamma = \frac{1}{40}$ is adiabatic index, M is molar weight in $\mathrm{kg/mol}$

Sound Intensity

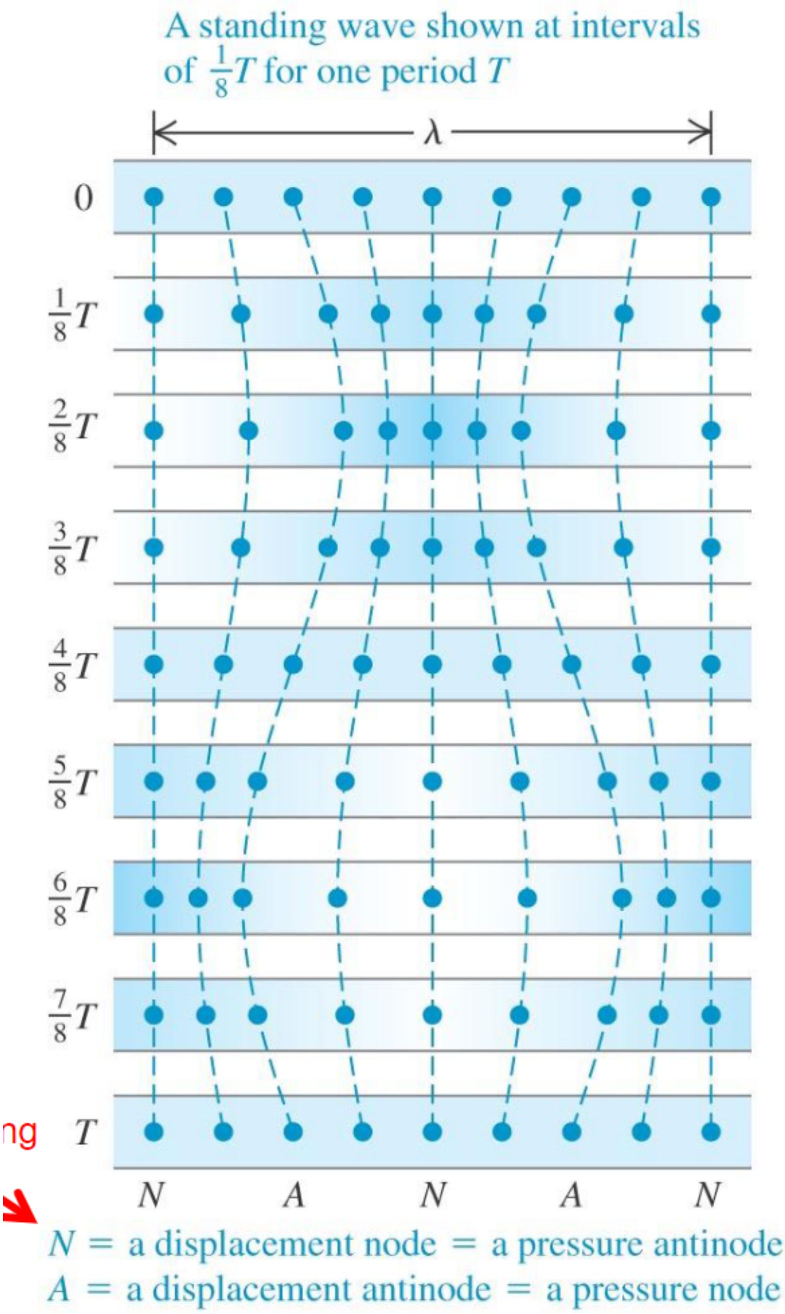
- Review of Intensity
- Definition: **energy / (time * area) = power / area**
- Unit: $\mathrm{W/m^2}$
- $I = \frac{1}{2}\sqrt{\rho B}\omega^2 A^2 = \frac{p_{\max}^2}{2\rho v} = \frac{p_{\max}^2}{2\sqrt{\rho B}}$
- $A \propto \frac{1}{\omega} \propto \frac{1}{f}$
- The sound waves are sent at a **hemisphere**, $\mathrm{Area} = \frac{1}{2} \times 4\pi R^2 = 2\pi R^2$
- Sound Intensity at distance R: $I = \frac{P}{w\pi R^2}$

Decibel Scale for Sound Intensity

- $\beta = (10\mathrm{dB}) \log \frac{I}{I_0}$
- $I_0 = 10^{-12}\mathrm{W/m^2}$
- $0\mathrm{dB}$ = threshold of hearing at $1000\mathrm{Hz}$
- $\beta_2 - \beta_1 = (10\mathrm{dB}) \log \frac{I_2}{I_1}$
- $\frac{I_2}{I_1} = \left(\frac{R_1}{R_2}\right)^2$

Standing Wave of Sound

- Sound is Longitudal
- Node = 0 Displacement = Max PressureΔ
- Antinode = Max Displacement = 0 PressureΔ
- Antinodes at $\frac{2n+1}{4}\lambda$
- **Don't here sound at pressure node = dp antinode =** $\frac{2n+1}{4}\lambda$



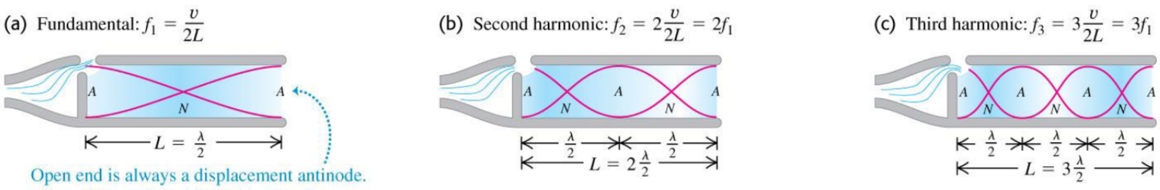
Organ Pipes

- **dp antinode** at mouth, because it's open end
- open pipe: $\lambda_n = \frac{2L}{n}$, $n+1$ antinodes and n nodes (total $2n+1$)

Harmonics in an open pipe

- An *open pipe* is open at both ends. Sound inside the pipe are reflected back into the pipe by the two open ends and a standing wave is formed.
- Open ends has displacement anti-nodes, as air molecules can move freely.
- For an open pipe: $n\lambda_n = 2L$

  $\lambda_n = 2L/n$ and $f_n = nv/2L$ $(n = 1, 2, 3, \ldots)$ (16.18)
- Figure below shows some harmonics in an open pipe.

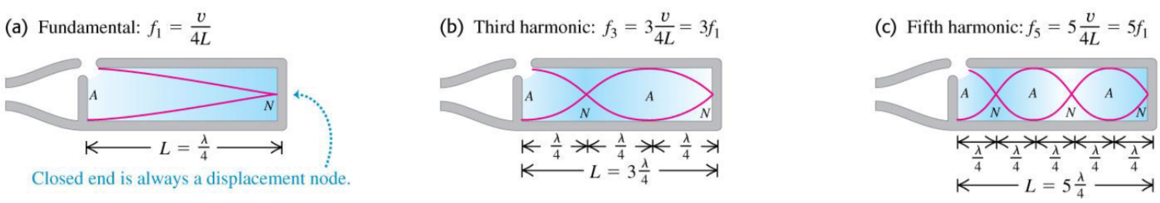


Harmonics in a stopped pipe

- A *stopped pipe* is open at one end and closed at the other end.
- For a stopped pipe, close end is displacement node and the open end is displacement antinode. So, $n\lambda_n /4 = L$

$$\boxed{\lambda_n = 4L/n \text{ and } f_n = nv/4L \ (n = 1, 3, 5, \ldots)\ (16.22)}$$

- Figure below shows some harmonics in a stopped pipe.

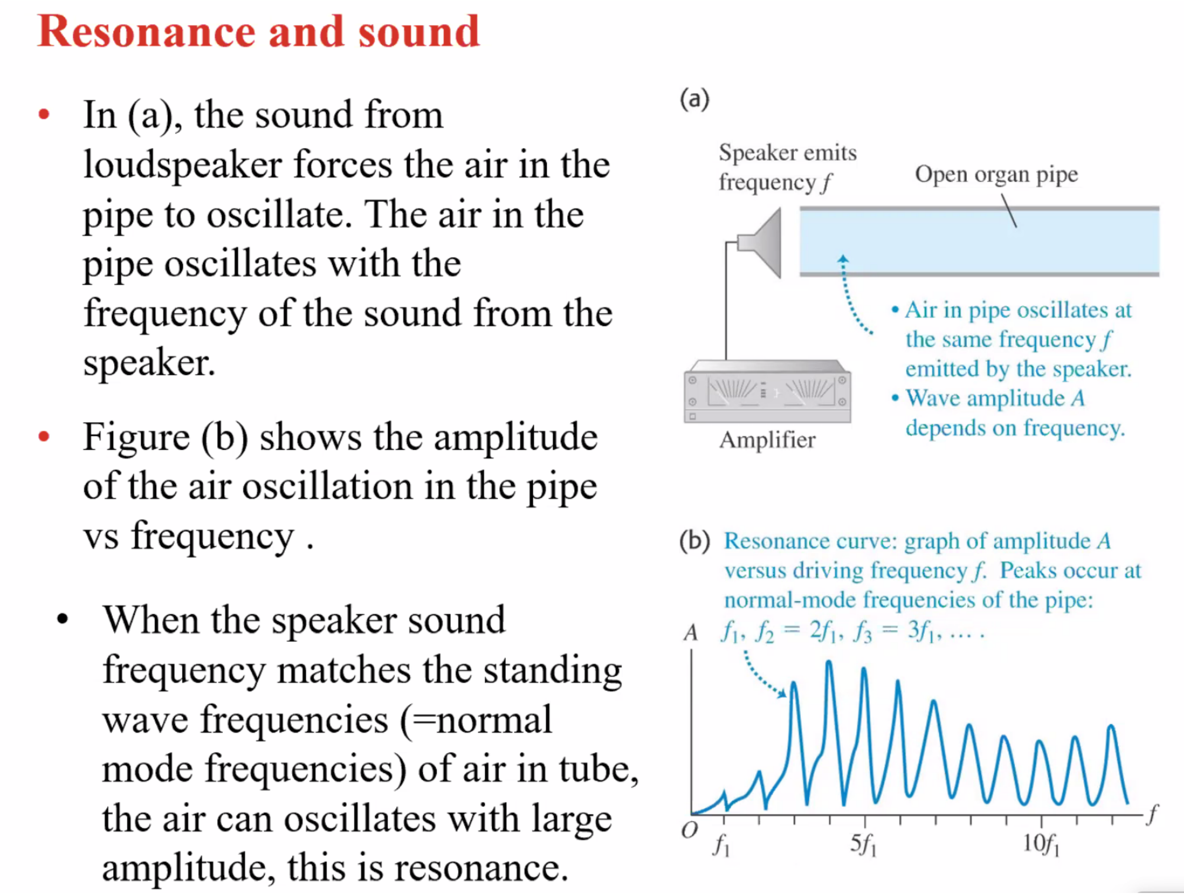


fundamental mode: $f_1 = v/4L$ (16.20)

- stopped pipe: $\lambda_{2n-1} = \frac{4L}{2n-1}$, n antinodes and n nodes (total $2n$s)
- **second overtone = 3rd possible harmonic = 3rd in open or 5th in stopped**
- remember: for **string fixed at both ends**, $f_1 = \frac{v}{2L}$

Resonance

- normal mode frequency $= 1, 2, 3, \cdots \times f_0$



Interference

- **difference in path lengths** (e.g. Δx between two loudspeakers and a point)
- **constructive interference**, $\Delta x = n\lambda, n \in \mathbb{N}$
- **destructive interference**, $\Delta x = \frac{(2n+1)\lambda}{2}, n \in \mathbb{N}$

Beats

- two tones of slightly different frequency
- $f_{\text{beat}} = f_a - f_b$
- (displacement) **frequency**=$\frac{\omega_a+\omega_b}{2}$
- **amplitude**=$\cos\left(\frac{\omega_a-\omega_b}{2}t\right) = \cos\left(\frac{2\pi(f_a-f_b)}{2}t\right)$
- **what we hear is intensity (sound energy) -> reflected by amplitude**
- $I \propto \cos^2\left(\frac{2\pi(f_a-f_b)}{2}t\right)$

$$\begin{aligned}
y(x,t) &= y_a + y_b \\
&= A\cos(k_a x - \omega_a t) + A\cos(k_b x - \omega_b t) \\
y(0,t) &= A\cos(\omega_a t) + A\cos(\omega_b t) \\
&= 2A\cos\left(\frac{\omega_a + \omega_b}{2}t\right)\cos\left(\frac{\omega_a - \omega_b}{2}t\right)
\end{aligned}$$

Doppler Effect

- Source towards listener $f_L = f_s \frac{v}{v - v_s}$
- Source away from listener $f_L = f_s \frac{v}{v + v_s}$
- Listener towards source $f_L = f_s \frac{v + v_L}{v}$
- Listener away from source $f_L = f_s \frac{v - v_L}{s}$

$$\frac{f_L}{f_s} = \frac{v \mp v_L}{v \pm v_s}$$

- **getting closer, higher frequency**
- **reflected sound**

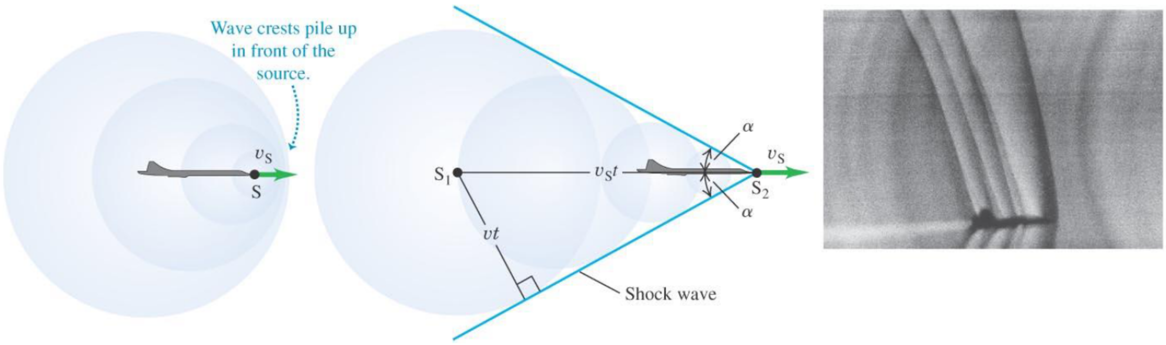
$$\frac{f_R}{f_s} = \frac{v}{v - v_s} \frac{v + v_s}{v} = \frac{v + v_s}{v - v_s}$$

- source speed = sound speed, **shock wave**
- source speed > sound speed, **sonic bloom** (supersonic)
- $\sin \alpha = \frac{v}{v_s}, \frac{v_s}{v} = \text{Mach Number}$ (example: airplane at Mach 1.75)

(a) Sound source S (airplane) moving at nearly the speed of sound

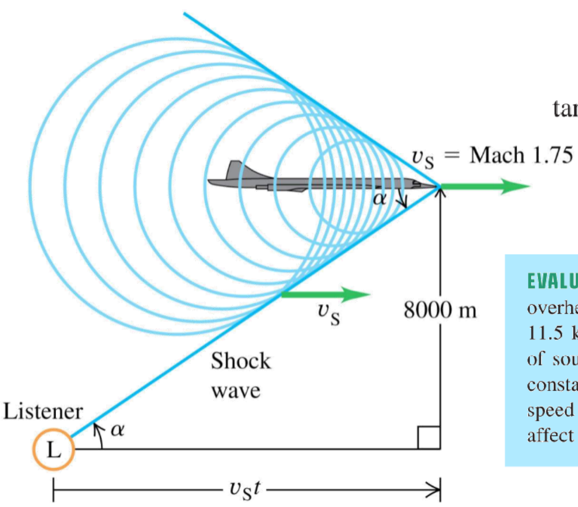
(b) Sound source moving faster than the speed of sound

(c) Shock waves around a supersonic airplane



Example 16.19: Sonic boom from a supersonic airplane

An airplane is flying at Mach 1.75 at an altitude of 8000 m, where the speed of sound is 320 m/s. How long after the plane passes directly overhead will you hear the sonic boom?



SOLUTION From Eq. (16.31) the angle α of the shock cone is

$$\alpha = \arcsin \frac{1}{1.75} = 34.8°$$

The speed of the plane is the speed of sound multiplied by the Mach number:

$$v_S = (1.75)(320\ \text{m/s}) = 560\ \text{m/s}$$

$$\tan \alpha = \frac{8000\ \text{m}}{v_S t}$$

$$t = \frac{8000\ \text{m}}{(560\ \text{m/s})(\tan 34.8°)} = 20.5\ \text{s}$$

EVALUATE: You hear the boom 20.5 s after the airplane passes overhead, at which time it has traveled $(560\ \text{m/s})(20.5\ \text{s}) = 11.5$ km since it passed overhead. We have assumed that the speed of sound is the same at all altitudes, so that $\alpha = \arcsin v/v_S$ is constant and the shock wave forms a perfect cone. In fact, the speed of sound decreases with increasing altitude. How would this affect the value of t?